Inquisitive Semantics

Ivano Ciardelli
Jeroen Groenendijk
Floris Roelofsen
Inquisitive Semantics
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Inquisitive Semantics

IVANO CIARDELLI, JEROEN GROENENDIJK, AND FLORIS ROELOFSEN
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General preface

*Oxford Surveys in Semantics and Pragmatics* aims to convey to the reader the life and spirit of the study of meaning in natural language. Its volumes provide distillations of the central empirical questions driving research in contemporary semantics and pragmatics, and distinguish the most important lines of inquiry into these questions. Each volume offers the reader an overview of the topic at hand, a critical survey of the major approaches to it, and an assessment of what consensus (if any) exists. By putting empirical puzzles and theoretical debates into a comprehensible perspective, each author seeks to provide orientation and direction to the topic, thereby providing the context for a deeper understanding of both the complexity of the phenomena and the crucial features of the semantic and pragmatic theories designed to explain them. The books in the series offer researchers in linguistics and related areas—including syntax, cognitive science, computer science, and philosophy—both a valuable resource for instruction and reference and a state-of-the-art perspective on contemporary semantic and pragmatic theory from the experts shaping the field.

In this volume, Ivano Ciardelli, Jeroen Groenendijk, and Floris Roelofsen provide the first comprehensive introduction to Inquisitive Semantics, a theory of linguistic meaning that aims to unify the analysis of declarative and interrogative sentences. Unlike traditional models of meaning, which distinguish declaratives and interrogatives either in terms of semantic type or in terms of force, Inquisitive Semantics builds an integrated analysis of both sentence types around a particular formalization of information states, called “issues.” In the first part of this book, Ciardelli, Groenendijk, and Roelofsen lay out the formal foundations of the theory, showing how it provides a general representation of semantic content and conversational contexts. In the second part, they demonstrate the generality of the framework by first applying it to the analysis of multiple types of interrogatives, extending it to the analysis of disjunction, conditionals and propositional attitudes, and finally comparing it to previous analyses of questions. With its clear exposition, detailed formalization, substantive discussion of empirical phenomena, and carefully constructed exercises in inquisitive
semantic analysis, this book provides newcomers to the framework with a much-needed introduction, and experienced researchers with a valuable resource for further exploring its applications.

Chris Barker  
*New York University*  
Christopher Kennedy  
*University of Chicago*
Acknowledgments

This book has grown out of lecture notes for courses at the European and North American Summer Schools in Logic, Language, and Information (ESSLLI 2015 and NASSLLI 2012), as well as yearly instalments of the course Logic and Conversation at the University of Amsterdam (2012–17). We are very grateful to the students and colleagues who attended these courses and provided insightful feedback. We are especially grateful to Lucas Champollion, Donka Farkas, and Anna Szabolcsi for comments on various parts of the book that led to considerable improvements.

Many people have contributed to the development of the framework presented here, as will be witnessed by numerous references throughout the book. In particular, Chapters 6 and 7 have grown out of close collaborations with Donka Farkas and with Lucas Champollion and Linmin Zhang, respectively. These collaborations have been instrumental in shaping the overall argument presented in the book.

We are very grateful to three anonymous OUP book reviewers for detailed, constructive feedback on the submitted version of the book manuscript, and to the OUP editorial staff for their help in the publication process.

Finally, we gratefully acknowledge financial support from the Netherlands Organization for Scientific Research (NWO, grant numbers 360-20-260 and 275-80-003) and the European Research Council (ERC, grant number 680220).
Sources

Many of the papers referred to in this book can be accessed through www.illc.uva.nl/inquisitivesemantics/papers. Some computational tools that might help the reader to become familiar with the framework presented in the book are available at www.illc.uva.nl/inquisitivesemantics/resources.

This book brings together a number of ideas and results from previous publications, manuscripts, and teaching materials. Below we list the main sources for each chapter, which in some cases contain more comprehensive discussion of the ideas presented here.

- Chapter 2: Ciardelli, Groenendijk, and Roelofsen (2013a)
- Chapter 3: Roelofsen (2013a)
- Chapter 4: Ciardelli (2009); Groenendijk and Roelofsen (2009); Ciardelli and Roelofsen (2011); Roelofsen (2013a); Ciardelli (2016d)
- Chapter 6: Roelofsen (2013c, 2015a); Roelofsen and Farkas (2015); Farkas and Roelofsen (2017)
- Chapter 7: Champollion, Ciardelli, and Zhang (2016); Ciardelli (2016b); Ciardelli, Zhang, and Champollion (2017c)
- Chapter 8: Ciardelli and Roelofsen (2015, 2018)
- Chapter 9: Ciardelli, Groenendijk, and Roelofsen (2013a); Ciardelli (2017b); Ciardelli and Roelofsen (2017a); Farkas and Roelofsen (2017)
1 Introduction

Inquisitive semantics is a new semantic framework mainly intended for the analysis of linguistic information exchange. Information exchange can be seen as a process of raising and resolving issues. Inquisitive semantics provides a new formal notion of issues, which makes it possible to model various concepts that are crucial for the analysis of linguistic information exchange in a more refined and more principled way than has been possible in previous frameworks. In particular:

1. The semantic content of both declarative and interrogative sentences can be represented in an integrated way, capturing not only the information that such sentences convey, but also the issues that they raise;
2. Similarly, conversational contexts can be modeled as encompassing not just the information that has been established in the conversation so far, but also the issues that have been brought up;
3. And finally, it becomes possible to formally represent a broader range of propositional attitudes that are relevant for information exchange: besides the familiar information-directed attitudes like knowing and believing, issue-directed attitudes like wondering can be captured as well.

This book provides a detailed exposition of the most basic features of inquisitive semantics, and demonstrates some of the advantages that the framework has with respect to previously proposed ways of representing semantic content, conversational contexts, and propositional attitudes.

This introductory chapter will proceed to argue in some detail why a framework like inquisitive semantics is needed for a satisfactory analysis of information exchange (Section 1.1), and will end with a global outline of the remaining chapters (Section 1.2).
1.1 Motivation

The most basic question that needs to be addressed in more detail before we introduce the new formal notion of issues that forms the cornerstone of inquisitive semantics is why such a notion is needed at all for the analysis of linguistic information exchange. This will be done in Section 1.1.1.

A second fundamental point that we want to make is that the analysis of linguistic information exchange does not just require a semantic theory of declaratives and another semantic theory of interrogatives side by side, but rather an \textit{integrated} theory of declaratives and interrogatives; neither sentence type can be fully understood in isolation. Reasons for this will be given in Section 1.1.2.

Finally, a third important point is that a semantic theory of declaratives and interrogatives should not employ two different notions of semantic content, one for declaratives and one for interrogatives, but should rather be based on a single notion of semantic content that is general enough to capture both the information that sentences convey and the issues that they may raise. This point will be substantiated in Section 1.1.3.

1.1.1 Why do we need a formal notion of issues?

There are several reasons why a formal notion of issues is needed for the analysis of linguistic information exchange, and each of these is related to one of the three aspects of information exchange listed above: some arise from the need for a suitable notion of semantic content, some from the need for a suitable model of conversational contexts, and yet others from the need for a sufficiently refined representation of the mental states of conversational participants. We will discuss each in turn.

\textbf{Reason 1: To represent the content of interrogative sentences}  The semantic content of a declarative sentence is standardly construed as a set of possible worlds, those worlds that are compatible with the information that the sentence conveys (as per the conventions of the language; additional information may be conveyed pragmatically when the sentence is uttered). This set of worlds is referred to as the \textit{proposition} that the sentence expresses.

This notion of semantic content works well for declarative sentences, whose main conversational role is indeed to provide information. For
instance, the main communicative function of the declarative sentence in (1) below is to convey the information that Bill is coming.

(1) Bill is coming.

But information exchange typically does not just consist in a sequence of declarative sentences. An equally important role is played by interrogative sentences, whose main conversational role is to raise issues.

Can the semantic content of an interrogative sentence be construed as a set of possible worlds as well? Consider the example in (2), a polar interrogative:

(2) Is Bill coming?

Frege (1918) famously proposed that the interrogative in (2) and the declarative in (1) can indeed be taken to have the same semantic content:

An interrogative sentence and an indicative one contain the same thought; but the indicative contains something else as well, namely, the assertion. The interrogative sentence contains something more too, namely a request. Therefore two things must be distinguished in an indicative sentence: the content, which it has in common with the corresponding sentence-question, and the assertion. (Frege, 1918, p. 294)

So the idea is that declaratives and interrogatives have the same semantic content—a proposition—but come with a different force—either assertion or request. This idea has been quite prominent in the literature, especially in speech act theory (Searle, 1969; Vanderveken, 1990). However, as noted by Frege himself, it is limited in scope. It may work for simple polar interrogatives, but not for many other kinds of interrogatives, like (3)–(4):

(3) Is Bill coming, or Sue?
(4) Who is coming?

Moreover, as has been argued extensively in the more recent literature (see especially Groenendijk and Stokhof, 1997), even the idea that a plain polar interrogative has the same content as the corresponding declarative is problematic. In particular, when applied to embedded cases it is not compatible with the principle of compositionality, which requires that the semantic content of a compound expression be determined by the semantic content of its constituent parts, and the way in

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1 The page reference is to the translated version, Frege (1956).
2 See also recent work on questions in dynamic epistemic logic (van Benthem and Minică, 2012).
which these parts are combined. To see this, compare the following two examples, which contain embedded variants of the declarative in (1) and the polar interrogative in (2), respectively:

(5) John knows that Bill is coming.
(6) John knows whether Bill is coming.

If the embedded clauses had the same content, then by the principle of compositionality the two sentences as a whole should also have the same content. But this is clearly not the case. So the embedded clauses must differ in content.

Thus, the standard notion of semantic content does not seem applicable to interrogative sentences. Rather, what we need for interrogatives is a notion of content that directly captures the issues that they raise.3

Reason 2: To model conversational contexts It has been argued extensively in the literature that conversational contexts have to be modeled in a way that does not only take account of the information that has been established in the conversation so far, but also of the issues that have been brought up, often referred to as the questions under discussion (Carlson, 1983; Groenendijk and Stokhof, 1984; van Kuppevelt, 1995; Ginzburg, 1996; Roberts, 1996; Büring, 2003; Beaver and Clark, 2008; Tonhauser et al., 2013, among others). We will briefly discuss two reasons why this is important. First, it is needed to develop a formal theory of pragmatic reasoning and the conversational implicatures that result from such reasoning. And second, it is needed for a theory of information structural phenomena like topic and focus marking. Let us first consider pragmatic reasoning.

A key notion in pragmatic reasoning is the notion of relevance. When is a contribution to a conversation relevant for the purposes at hand? One natural answer is that a contribution is relevant just in case it addresses one of the issues under consideration. Even if the issues under consideration only partially characterize what is ‘relevant’ in a broader sense, this partial characterization is crucial for a formal theory of conversational implicatures. For, the issues under consideration influence which conversational implicatures arise. To see this, consider the following examples:

3 There is an extensive literature on the semantics of interrogatives (Hamblin, 1973; Karttunen, 1977; Groenendijk and Stokhof, 1984, among many others), and inquisitive semantics strongly builds on the insights that have emerged from this work. A detailed comparison will be provided in Chapter 9.
B’s utterance is exactly the same in both cases, but the issue that it addresses is different. As a result, in (7), where the question under discussion is what B did this morning, there is a conversational implicature that B did not do anything besides reading the newspaper, i.e., that he did not do the laundry for instance. On the other hand, in (8), where the question under discussion is what B read this morning, there is a weaker conversational implicature, to the effect that B did not read anything besides the newspaper. This does not imply that he did not do other things, such as the laundry. Thus, we see that pragmatic reasoning is sensitive to the issues that are at play in the context of utterance.

Now let us illustrate the importance of contextual issues for information structural phenomena. We will concentrate on focus marking. Languages generally have grammatical ways to signal which part of a sentence is in focus and which part is backgrounded. In English, the focus/background distinction is marked intonationally: focused constituents receive prominent pitch accents, while backgrounded constituents do not. In other languages, focus is sometimes marked by means of special particles or by means of word order.

Which constituents should be marked as being in focus and which should be marked as being backgrounded is determined, at least partly, by the issue that is being addressed. To see this, consider the following examples, where capitalization is used to indicate focus marking by means of prominent pitch accents.

(9) A: Who did Alf rescue?
   B: Alf rescued BEA. / #ALF rescued Bea.

(10) A: Who rescued Bea?
     B: ALF rescued Bea. / #Alf rescued BEA.

If the question is who Alf rescued, as in (9), then the response that Alf rescued Bea must be pronounced with a prominent pitch accent on Bea. Placing a pitch accent on Alf instead results in infelicity. On the other hand, if the question is who rescued Bea, as in (10), then the same response, i.e., that Alf rescued Bea, must be pronounced with a prominent pitch accent on Alf rather than Bea. Thus, we see that focus
marking, just like pragmatic reasoning, is sensitive to the issue under discussion.4

Reason 3: To model issue-directed propositional attitudes and capture the meaning of verbs that report such attitudes  In order to understand linguistic information exchange, it is important to have a way of representing the information that is available to the agents participating in the exchange, as well as the issues that they are interested in. In other words, we need to be able to model what the agents know or believe at any given time, and also what they wonder about. Knowledge and belief are information-directed propositional attitudes; wondering is an issue-directed propositional attitude. The standard way to model the knowledge and beliefs of an agent is as a set of possible worlds, namely those worlds that are compatible with what the agent knows or believes. Such a set of worlds is thought of as representing the agent’s information state. Similarly, in order to capture what an agent wonders about, we need a representation of her inquisitive state. For such a representation, we again need a formal notion of issues.

Moreover, turning back to language, just like there are verbs like know and believe that describe the information state of an agent, as in (11) below, there are also verbs like wonder and be curious that describe the inquisitive state of an agent, as in (12).

(11) John knows that Bill is coming.
(12) John wonders who is coming.

Clearly, in order to analyse the meaning of verbs like wonder we do not only need a suitable representation of the content of the interrogative clause that the verb takes as its complement (here, who is coming), but also a suitable representation of the inquisitive state of the subject of the verb (here, John).

1.1.2 Declaratives and interrogatives cannot be treated separately

The analysis of linguistic information exchange requires a semantic theory of declaratives and one of interrogatives. A question that naturally arises, then, is whether the two sentence types could be analysed separately, or whether a more integrated approach is called for. Below we

4 Besides pragmatic reasoning and information structural phenomena like topic and focus marking, it has been argued that a model of conversational contexts that comprises the issues that have been raised is also needed for a suitable analysis of discourse particles (see, e.g., Rojas-Esponda, 2013) and presupposition projection (e.g., Tonhauser et al., 2013).
give two reasons why neither declaratives nor interrogatives can be fully understood in isolation, making an integrated approach necessary.

**Reason 1: Mutual embedding** Declarative and interrogative sentences can be embedded into one another, as exemplified in (13)–(15).

(13) Bill asked me who won. embedded interrogative
(14) Who told you that Jane won? embedded declarative
(15) Bill asked me who told you that Jane won.

So the meaning of a declarative sentence is sometimes partly determined by the meaning of an embedded interrogative sentence, and vice versa. Clearly, then, a complete semantic account of declaratives cannot be achieved without getting a handle on interrogatives, and the other way around, a complete semantic account of interrogatives is impossible without a treatment of declaratives. Thus, the two have to be analysed hand in hand; considering them in isolation is bound to lead to incomplete theories.

**Reason 2: Interpretational dependencies** As illustrated in (16) and (17), the interpretation of a declarative sentence sometimes partly depends on the issue raised by a preceding interrogative. Notice that examples (16)–(17) differ from the previous examples (7)–(8) in that they contain the particle *only*.

(16) A: What did you do this morning?  
   B: I only read the newspaper.  \( \sim B \) did not do the laundry
(17) A: What did you read this morning?  
   B: I only read the newspaper.  \( \not\sim B \) did not do the laundry

If the question is what you *did* this morning, as in (16), then the truth of the statement that you only read the newspaper requires that you did not do other things, such as the laundry. On the other hand, if the question is what you *read* this morning, as in (17), then the truth of the statement that you only read the newspaper just requires that you did not read anything else, while it is compatible with the fact that you did do other things besides reading, such as the laundry. Thus, not just the pragmatic implicatures that a declarative statement may induce, but even its truth-conditional content can depend on the issue that is addressed, which again means that analyzing declaratives in isolation,
without taking interrogatives into account as well, is bound to lead to an incomplete theory.

1.1.3 Why do we need an integrated notion of semantic content?

As we discussed above, the notion of semantic content that is commonly assumed for declarative sentences does not seem suitable for interrogative sentences. In principle, this does not mean that there is anything wrong with this standard notion. We could attempt to construe a suitable notion of content for interrogatives, and maintain the existing notion for declaratives. This, indeed, is the approach that has been taken in most previous work (see Groenendijk and Stokhof, 1997, for an overview). We will argue, however, that a single, integrated notion of semantic content is to be preferred.

**Reason 1: Common building blocks** Declaratives and interrogatives are to a large extent built up from the same lexical, morphological, and intonational elements. Clearly, we would like to have a uniform semantic account of these elements, i.e., an account that captures their semantic contribution in full generality, rather than two separate accounts, one capturing their semantic contribution when they are part of declarative sentences and the other when they are part of interrogative sentences.

To make this concrete, consider the following two examples, a declarative and an interrogative which are built up from exactly the same lexical items and also exhibit the same intonation pattern (we use ↑ and ↓ to indicate rising and falling intonation, respectively).

(18) Luca is from Italy↑ or from Spain↓.
(19) Is Luca from Italy↑ or from Spain↓?

In uttering the declarative in (18), a speaker provides the information that Luca is from Italy or from Spain, and she does not request any further information from other conversational participants. On the other hand, in uttering the interrogative in (19), she takes the information that Luca is from Italy or Spain for granted, and requests other participants to provide further information determining exactly which of the two countries he is from.

Both sentences contain the disjunction word or. In declaratives, or is normally taken to yield the union of the semantic values of the disjuncts. In (18), each disjunct expresses a proposition, standardly represented as
1.1 MOTIVATION

a set of possible worlds: the semantic value of the first disjunct is the set of worlds where Luca is from Italy, and the semantic value of the second disjunct is the set of worlds where Luca is from Spain. The proposition expressed by (18) is the union of these two sets, i.e., the set of all worlds where Luca is from either country.

This seems a reasonable account of or in declaratives. But what is the role of or in interrogatives? Ultimately, we would like to have an account of or that is general enough to capture its semantic contribution in both declaratives and interrogatives in a uniform way. Assuming different notions of semantic content for declarative and interrogative sentences constitutes an obstacle for such a uniform account. By contrast, as we will see, such an account naturally comes within reach once we assume an integrated notion of semantic content. In this approach, the semantic content of a complete sentence should capture both the information that the sentence conveys and the issue that it raises (where of course, either may be trivial), and the semantic content of any sub-sentential constituent should capture the contribution that this constituent makes both to the information conveyed and to the issue raised by the sentence.

**Reason 2: Entailment**  Entailment is normally thought of as a logical relation between declarative sentences. One sentence is taken to entail another if the first conveys at least as much information as the second. This logical relation plays a central role in the standard logical framework for natural language semantics. For one thing, predictions about entailment constitute one of the primary criteria for empirical success of semantic theories. That is, a theory is assessed by testing its predictions about entailment. But besides this, entailment is important in various other respects as well. For instance, it plays a crucial role in the derivation of quantity implicatures, which involves comparing the sentence that a speaker actually uttered with other sentences that the speaker could have uttered instead. This comparison is done in terms of informative strength, which is captured by entailment (see Grice, 1975, and much subsequent work). Similarly, entailment is needed to formulate interpretive principles like the Strongest Meaning Hypothesis, which has been argued to play a crucial role in the resolution of semantic underspecification, for instance in the interpretation of plural predication (Dalrymple et al., 1998; Winter, 2001). And as a final example, entailment has been used to characterize the distribution of positive and negative polarity items in terms of upward and downward
entailing environments (e.g., Ladusaw, 1980; Kadmon and Landman, 1993).

Clearly, we would like our theories of quantity implicatures, plural predication, polarity items, etc., to apply in a uniform way to declarative and interrogative constructions. However, since the standard notion of entailment compares two sentences in terms of their informative, truth-conditional content (and sub-sentential expressions in terms of their contribution to the informative content of the sentences that they are part of), it does not suitably apply to interrogatives. For this reason, the scope of entailment-based theories such as the ones just mentioned is currently restricted to declaratives.

What we need, then, is a notion of entailment that is general enough to apply to both declaratives and interrogatives in a uniform way. We expect, for instance, to be able to account in a uniform way for the fact that the declarative in (20a) entails the one in (20b), and for the fact that the interrogative in (21a) entails the one in (21b).

(20) a. The number of planets is 8.
   b. The number of planets is even.

(21) a. What is the number of planets?
   b. Is the number of planets even?

For this, we need a notion of entailment which is sensitive to both informative and inquisitive strength. Such a notion can be naturally defined once we operate with a notion of semantic content that encompasses both informative and inquisitive content.

**Reason 3: Logical operations**  Two declarative sentences can be combined by means of conjunction and disjunction.

(22) Peter rented a car and Mary booked a hotel.
(23) Peter rented a car or he borrowed one.

This does not only hold for root declaratives, but also for embedded ones.

(24) I believe that Peter rented a car and that Mary booked a hotel.
(25) I believe that Peter rented a car or that he borrowed one.

This is also true for interrogatives, both embedded and unembedded ones.\(^5\)

\(^5\) While the possibility of conjoining interrogative sentences is uncontroversial, the possibility of disjoining interrogatives has been disputed by Szabolcsi (1997, 2015a) and Krifka (2001b). In Section 9.2.2 we will examine Szabolcsi’s argument in some detail. On
(26) Where can we rent a car, and which hotel should we take?
(27) Where can we rent a car, or who might have one that we could borrow?
(28) I’m investigating where we can rent a car and which hotel we should take.
(29) I’m investigating where we can rent a car or who might have one that we could borrow.

These parallels between declaratives and interrogatives exist not only in English, but in many other languages as well: words that are used to conjoin declaratives are also used to conjoin interrogatives, and words that are used to disjoin declaratives can often also be used to disjoin interrogatives.

What we would like to have, then, is an account of conjunction and disjunction that does not just apply to declaratives, but that is general enough to apply to both declaratives and interrogatives in a uniform way. As we will see, such an account comes within reach if we analyse declaratives and interrogatives by means of a single notion of semantic content that encompasses both informative and inquisitive content.

Besides conjunction and disjunction, another logical operation that can be performed both on declaratives and on interrogatives is conditionalization, as exemplified in (30) and (31).

(30) If Bill asks Mary out, she will accept.
(31) If Bill asks Mary out, will she accept?

This calls for an account of conditionals that applies uniformly, regardless of whether the consequent is a declarative or an interrogative sentence. Again, such an account is facilitated by a semantic framework which encompasses both informative and inquisitive content.

1.2 Main aims and outline of the book

Given the above considerations, our main high-level aims in this book will be to introduce:

1. A formal notion of issues that allows for a suitable representation of semantic content, conversational contexts, and propositional attitudes;

the basis of examples such as (27) and (29), we will argue that disjoining interrogatives is in principle possible, and that the meaning of the resulting disjunction is correctly derived by applying inquisitive disjunction to the meanings of the two interrogative disjuncts.
2. A logical framework that allows for an integrated semantic analysis of declarative and interrogative sentences, with a single notion of semantic content which is general enough to deal with both sentence types at once, rather than a separate notion of semantic content for each sentence type.

The remaining chapters of the book broadly fall into two parts. The first part, spanning Chapters 2–4, provides a detailed exposition of the basic inquisitive semantics framework. The second part, consisting of Chapters 5–9, discusses several applications of the framework and compares it to previous work.

More specifically, Chapter 2 introduces the new notions of issues, propositions, and conversational contexts that form the heart of inquisitive semantics; Chapter 3 identifies the basic operations that can be performed on inquisitive propositions; and Chapter 4 presents an inquisitive semantics for the language of first-order logic.

Then, turning to the second part, Chapter 5 shows how the meaning of various kinds of questions occurring in natural languages can be captured in the framework developed in Chapters 2–4; Chapter 6 shows how to derive the meaning of various declarative and interrogative sentence types in a compositional way, providing a concrete illustration of the benefits of treating informative and inquisitive content in an integrated way; Chapter 7 argues that the truth-conditions of certain declarative sentences—in particular, conditionals—depend on the inquisitive content of their constituents, which shows that the richer notion of semantic content that inquisitive semantics provides is beneficial even if one is just concerned with declaratives; Chapter 8 discusses the representation of information-directed and issue-directed propositional attitudes, as well as the semantics of verbs like know and wonder which are used to report such attitudes; and Chapter 9 discusses the advantages of inquisitive semantics as a framework for the semantic analysis of interrogatives in comparison with previous work. Finally, Chapter 10 concludes with a schematic overview of the book, and discusses to what extent it meets the two high-level desiderata listed at the beginning of the section.

The Further Reading section at the back of the book provides some pointers to work that further extends or applies the framework presented here.
Basic notions

In the previous chapter we have argued that a formal notion of *issues* is of crucial importance for the analysis of linguistic information exchange. The present chapter specifies how issues are formally defined in inquisitive semantics. It also defines three other basic notions—*information states, propositions*, and *conversational contexts*—and a number of fundamental relations that may hold between such entities. In particular, as depicted in Figure 2.1, we will specify what it means for an information state to *resolve* an issue or to *support* a proposition, what it means for a context to be *updated* with a proposition, when one context is an *extension* of another, when one proposition *entails* another, when one information state is an *enhancement* of another, and when one issue is a *refinement* of another.

Before turning to the inquisitive setting, however, we first briefly review how these notions—with the exception of issues—are standardly defined.

2.1 The standard picture

The simplest way to construe information states, propositions, and conversational contexts is as *sets of possible worlds* (see, e.g., Hintikka, 1962; Stalnaker, 1978). A set of possible worlds can be thought of as representing a certain *body of information*, namely the information that the actual world corresponds to one of the worlds in the set. Such a body of information may be seen as the information available to a certain conversational participant; in that case it can be taken to represent the information state of that participant. On the other hand, a body of information may also be seen as the information conveyed by a certain sentence; in that case it can be taken to constitute the semantic content of that sentence, the proposition that it expresses. And finally, a body of information could be seen as the information that has so far been commonly established by all the participants in a conversation; in
that case it embodies the common ground of the conversation, which constitutes a minimal representation of the conversational context.\(^1\)

Thus, depending on the perspective one takes, one and the same type of formal object—a set of possible worlds—can be used to model all three basic notions.

Let us now turn to the notions of enhancement (between information states), entailment (between propositions), and extension (between contexts). One information state \(s\) is an enhancement of another information state \(s'\) just in case all the information available in \(s'\) is also available in \(s\), i.e., if every candidate for the actual world that is ruled out by \(s'\) is also ruled out by \(s\). This holds just in case \(s \subseteq s'\). Similarly, one proposition \(p\) entails another proposition \(p'\) if and only if \(p\) contains at least as much information as \(p'\) does, i.e., if \(p \subseteq p'\), and one context \(c\) is an extension of another context \(c'\) if and only if all the information that is commonly established in \(c'\) is also commonly established in \(c\), i.e., if \(c \subseteq c'\). Thus, enhancement, entailment, and extension again formally all amount to the same relation, i.e., set inclusion, though in each case we take a somewhat different perspective on what this formal relation encodes, mirroring the different perspectives on sets of possible worlds.

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\(^1\) Sometimes a distinction is made between the common ground of a conversation and the context set (Stalnaker, 1978). The common ground is then construed as the set of pieces of information that are publicly shared among the conversational participants, and the context set as the set of possible worlds that are compatible with all these pieces of information. For our purposes, it will not be necessary to make this distinction, so we simply construe the common ground as the set of possible worlds that are compatible with the commonly established body of information.
when viewed as information states, propositions, and conversational contexts.

Now let us turn to the notion of support, which relates information states to propositions. An information state \( s \) is standardly taken to support a proposition \( p \) just in case the information embodied by \( p \) is already available in \( s \), i.e., if every candidate for the actual world that is ruled out by \( p \) is ruled out by \( s \) as well. This holds just in case \( s \subseteq p \). So support, just like entailment, enhancement, and extension, formally amounts to set inclusion.

Finally, let us consider the notion of update. The result of updating a context \( c \) with a proposition \( p \) is a new context \( c[p] \) which, besides the information already present in \( c \), also contains the information embodied by \( p \). That is, a candidate for the actual world is ruled out by \( c[p] \) if it was already ruled out by the information established in the old context \( c \), or if it is ruled out by the new information embodied by \( p \). Formally, this means that update amounts to set intersection: \( c[p] = c \cap p \).

What we have just reviewed is the simplest possible way to define information states, propositions, conversational contexts, and the relations that may hold between them in possible world semantics. Various more fine-grained versions of these basic notions have been proposed in the literature. Our goal here, however, is to construct the direct counterparts of these basic notions, together with a new notion of issues, in the inquisitive setting. Once these elementary notions are in place, one could set out to adapt the various refinements that have been proposed in the standard setting to the inquisitive setting as well. This will not be our direct concern in this book, but we will point to other work where such refinements have been pursued.

We are now ready to start building up the inquisitive semantics framework, starting with the notion of information states.

### 2.2 Information states

Information states are modeled in inquisitive semantics just as they are in the standard setting, namely as sets of possible worlds—those worlds that are compatible with the information available in the state. There is no need to change the notion of information states since—unlike in the case of propositions and conversational contexts, as we will see in Sections 2.4 and 2.5—this notion is just supposed to capture a body of information, and not anything issue-related.
Even though we straightforwardly adopt the standard notion of information states, we will define, discuss, and exemplify the notion somewhat more explicitly here than in the brief review in 2.1, in preparation of what is to come next. We use $W$ to denote the entire logical space, i.e., the set of all possible worlds.

**Definition 2.1 (Information states)**

An information state $s$ is a set of possible worlds, i.e., $s \subseteq W$.

We will often refer to information states simply as *states*. Figure 2.2 depicts some examples of information states in a logical space consisting of just four possible worlds: $w_1, w_2, w_3, w_4$. Intuitively, an information state can be thought of as locating the actual world within a certain region of the logical space. For instance, the state in Figure 2.2(d) contains the information that the actual world is located in the upper left corner of the logical space, while the state in Figure 2.2(c) contains the information that the actual world is located in the upper half of the logical space.

If $s$ and $t$ are two information states and $t \subseteq s$, then $t$ contains at least as much information as $s$; it locates the actual world with at least as much precision. In this case, we call $t$ an *enhancement* of $s$.

**Definition 2.2 (Enhancements)**

A state $t$ is called an enhancement of $s$ just in case $t \subseteq s$.

Note that we do not require that $t$ is *strictly* contained in $s$, i.e., that it contains strictly more information than $s$. If $t \subset s$, then we say that $t$ is a *proper* enhancement of $s$.

The four information states depicted in Figure 2.2 are arranged from left to right according to the enhancement order. The state in Figure 2.2(b) is an enhancement of the state in Figure 2.2(a), and so on. The state consisting of all possible worlds, $W$, depicted in Figure 2.2(a), is the least informed of all information states: any possible world is still taken to be a candidate for the actual world, which means that we have

**Figure 2.2** Information states.
2.3 Issues

We now turn to the notion of issues, in a sense the most central notion in inquisitive semantics. How should issues be represented formally? Our proposal is to characterize issues in terms of what information it takes to resolve them. That is, an issue is identified with a set of information states: those information states that contain enough information to resolve the issue.

We assume that every issue can be resolved in at least one way, which means that issues are identified with non-empty sets of information states. Moreover, if a certain state \( s \) contains enough information to resolve an issue \( I \), then this must also hold for every enhancement \( t \subseteq s \). This means that issues are always downward closed: if \( I \) contains a state \( s \), then it contains every \( t \subseteq s \) as well. Thus, issues are defined as non-empty, downward closed sets of information states.\(^2\)

**Definition 2.3 (Issues)**

An issue is a non-empty, downward closed set of information states.

**Definition 2.4 (Resolving an issue)**

We say that an information state \( s \) resolves an issue \( I \) just in case \( s \in I \). If \( s \) resolves \( I \), we will sometimes also say that \( I \) is settled in \( s \).

Figure 2.3 displays some issues. In order to keep the figures neat, only the maximal elements of these issues are depicted. Since issues are downward closed, we know that all enhancements of these maximal elements are also included in the issues at hand. The issue depicted in subfigure (a) can only be settled consistently by specifying precisely which world is the actual one. The issue depicted in subfigure (b) can

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\(^2\) Notice that this means that the inconsistent information state, \( \emptyset \), is an element of every issue. Thus, it is assumed that every issue is resolved in the inconsistent information state. This limit case may be regarded as a generalization of the *ex falso quodlibet* principle to issues.
be settled either by establishing that the actual world is an element of the set \(\{w_1, w_2\}\), or by establishing that it is an element of \(\{w_3, w_4\}\). The issue depicted in subfigure (c) can be settled either by establishing that the actual world is an element of \(\{w_1, w_3, w_4\}\), or by establishing that it is an element of \(\{w_2, w_3, w_4\}\). Finally, the issue in subfigure (d) is trivially settled: it does not require any information as to what the actual world may be.

Given an issue \(I\), the information state \(s := \bigcup I\) (the union of all the elements in \(I\)) contains exactly the information that is necessary and sufficient to guarantee that \(I\) can be truthfully resolved, i.e., to guarantee that there is an information state that resolves the issue and contains the actual world. For, if the actual world is located in \(s\), this means that it belongs to some \(t \in I\): in this case \(t\) is an information state that resolves \(I\) and contains the actual world, which means that \(I\) can indeed be truthfully resolved. On the other hand, if the actual world lies outside of \(s\), then it does not belong to any information state that resolves \(I\), which means that \(I\) cannot be resolved truthfully.

We think of the information state \(\bigcup I\) as capturing the information assumed by the issue \(I\), and we say that \(I\) is an issue over \(\bigcup I\).

**Definition 2.5 (Issues over a state)**

Let \(I\) be an issue and \(s\) an information state. Then we say that \(I\) is an issue over \(s\) if and only if \(\bigcup I = s\).

The issues depicted in Figure 2.3 are all issues over the information state \(W = \{w_1, w_2, w_3, w_4\}\). Notice that an issue \(I\) over a state \(s\) may contain \(s\) itself. In this case resolving \(I\) does not require any information beyond the information that is already available in \(s\). If so, we call \(I\) a trivial issue over \(s\). Downward closure implies that for any state \(s\) there is precisely one trivial issue over \(s\), namely the issue consisting of all enhancements of \(s\), i.e., the powerset of \(s\), which we denote as \(\wp(s)\). On the other hand, if
$I$ is an issue over $s$ and $s \not\in I$, then in order to settle $I$ further information is required, that is, a proper enhancement of $s$ must be established. In this case we call $I$ a proper issue over $s$. The issue in Figure 2.3(d) is the trivial issue over $W$; all the other issues in Figure 2.3 are proper issues over $W$.

Two issues over a state $s$ can be compared in terms of what it takes for them to be settled: one issue $I$ is at least as inquisitive as another issue $J$ just in case any state that settles $I$ also settles $J$. In this case we also say that $I$ is a refinement of $J$. Since an issue is identified with the set of states that settle it, the refinement order on issues just amounts to inclusion.

**Definition 2.6 (Issue refinement)**

Let $I, J$ be two issues over a state $s$. Then $I$ is at least as inquisitive as $J$ if and only if $I \subseteq J$. In this case we say that $I$ is a refinement of $J$.

Among the issues over a state $s$ there is always a least and a most inquisitive one. The least inquisitive issue over $s$ is the trivial issue $\wp(s)$, whose resolution, as we saw, requires no information beyond the information already available in $s$. The most inquisitive issue over $s$ is $\{\{w\} | w \in s\} \cup \{\emptyset\}$, which can only be settled consistently by providing a complete description of what the actual world is like. Among the issues in Figure 2.3, the issue in subfigure (a) is the most inquisitive issue over $W$, and thus a refinement of all other issues; the issue in subfigure (d) is the least inquisitive issue over $W$, and all other issues are refinements of it. As for the issues in subfigures (b) and (c), neither is a refinement of the other.

Suppose that a given information state $s$ resolves an issue $I$, and there is no weaker information state $t \supset s$ that also resolves $I$. Then $s$ contains just enough information to resolve $I$, it does not contain any superfluous information. Such information states are precisely the maximal elements of $I$, since information states consisting of more worlds contain less information. We will refer to these maximal elements as the alternatives in $I$.

**Definition 2.7 (Alternatives in an issue)**

The maximal elements of an issue $I$ are called the alternatives in $I$.

If an issue $I$ over a state $s$ is trivial, i.e., if $s \in I$, then $s$ is the unique maximal element of $I$, i.e., the unique alternative in $I$. On the other hand, if $I$ contains two or more alternatives, then it must be a non-trivial
issue. If an issue contains only finitely many information states, which will be the case in all the examples that we will consider here, then the connection between containing multiple alternatives and being a non-trivial issue also holds in the other direction.\footnote{To see that this does not generally hold for issues containing infinitely many states, consider an issue $I$ that contains an infinite chain of states, $s_1 \subset s_2 \subset s_3 \subset \ldots$, without any maximal element. Such an issue is non-trivial, since $\bigcup I \not\in I$, but it does not contain any alternatives. So, if an issue contains at least two alternatives, then it is always non-trivial, but the reverse implication only holds if we restrict ourselves to finite cases (Ciardelli, 2009).}

**Fact 2.8 (Multiple alternatives and proper issues)**

An issue containing finitely many elements is non-trivial if and only if it contains at least two alternatives.

This fact makes it very easy to see whether an issue is inquisitive, given a visual representation of it. For instance, the issue in Figure 2.3(d) is not inquisitive because it contains a single alternative, while all the other issues in Figure 2.3 are inquisitive because they contain multiple alternatives.

Note that, as exemplified in Figure 2.3(c), two alternatives in an issue $I$ may very well overlap, they do not have to be mutually exclusive. However, since alternatives are defined as maximal elements, one alternative can never be fully contained in another.\footnote{Our use of the term alternatives here is closely related to its use in the framework of alternative semantics (cf., Hamblin, 1973; Kratzer and Shimoyama, 2002; Simons, 2005; Alonso-Ovalle, 2006; Aloni, 2007, among others). One difference, however, is that in alternative semantics one alternative may very well be fully contained in another. We will discuss the commonalities and differences between inquisitive semantics and alternative semantics in more depth in Section 4.8 and Section 9.1.}

### 2.4 Propositions

Traditionally, the semantic content of a sentence, the proposition that it expresses, is intended to capture the information that a speaker conveys in asserting the sentence (as per the conventions of the language; additional information may be conveyed through pragmatic implicatures). In inquisitive semantics, propositions are not just intended to capture the information that is conveyed in uttering a sentence, but also the issue that may be raised in doing so. In short, propositions are intended to embody both informative and inquisitive content.

How should such more versatile propositions be modeled formally? The most straightforward option would be to construe a proposition...
as a pair \((\text{info}_P, \text{issue}_P)\), where \(\text{info}_P\) is a classical proposition, i.e., a set of possible worlds, embodying the informative content of \(P\), and \(\text{issue}_P\) an issue, embodying the inquisitive content of \(P\). We can then think of a speaker who utters a sentence expressing the proposition \(P\) as (i) providing the information represented by \(\text{info}_P\), and (ii) raising the issue represented by \(\text{issue}_P\). By the latter we mean that the speaker proposes to enhance the current common ground of the conversation in such a way that it comes to settle \(\text{issue}_P\); for short, we will say that the speaker ‘steers the common ground towards a state in \(\text{issue}_P\).

This notion of propositions is a natural starting point, but note that it does not impose any constraints on how the two components of a proposition should be related to each other. There are two constraints that we think should be enforced. First, all states that resolve \(\text{issue}_P\) should be enhancements of \(\text{info}_P\). It would not make sense for a speaker to steer the common ground towards a state where the information embodied by \(\text{info}_P\) itself is not commonly established. Formally, this means we should have that \(\bigcup \text{issue}_P \subseteq \text{info}_P\).

Second, the information that a speaker conveys should ensure that the issue she raises can be resolved truthfully. This means that we should have that \(\text{info}_P \subseteq \bigcup \text{issue}_P\). To see this, suppose that \(\text{info}_P \nsubseteq \bigcup \text{issue}_P\). Then there is a world \(w \in \text{info}_P\) which is not contained in any state that resolves \(\text{issue}_P\). According to \(\text{info}_P\), \(w\) may well be the actual world. Now, suppose it \emph{is} the actual world. Then there is no way of resolving \(\text{issue}_P\) without discarding the actual world. So, in this case \(\text{info}_P\) does not ensure that \(\text{issue}_P\) can be resolved truthfully.

Putting the two constraints together, we get that \(\text{issue}_P\) should be an issue \emph{over} \(\text{info}_P\): \(\bigcup \text{issue}_P = \text{info}_P\). Given this, our formal notion of propositions can be simplified considerably. After all, since \(\text{info}_P\) can always be retrieved from \(\text{issue}_P\), it can just as well be left out of the representation of \(P\). Thus, a proposition \(P\) can simply be represented as a non-empty, downward closed set of information states. The informative content of \(P\) is then represented by the union of all these states, \(\bigcup P\), while the issue embodied by \(P\) is the one which is resolved in a state \(s\) just in case \(s \in P\).

**Definition 2.9** (Propositions)

- A proposition \(P\) is a non-empty, downward closed set of information states.
- The set of all propositions will be denoted by \(\mathcal{P}\).
Definition 2.10 (Informative content)
For any proposition \( P \): \( \text{info}(P) := \bigcup P \)

Definition 2.11 (The issue embodied by a proposition)
The issue embodied by a proposition \( P \) is the one that is resolved in a state \( s \) just in case \( s \in P \).

2.4.1 Truth and support
We say that a proposition \( P \) is true in a world \( w \) just in case \( w \) is compatible with the informative content of \( P \), i.e., \( w \in \text{info}(P) \).

Definition 2.12 (Truth)
A proposition \( P \) is true in a world \( w \) just in case \( w \in \text{info}(P) \).

We say that an information state \( s \) supports a proposition \( P \) just in case it implies the informative content of \( P \), i.e., \( s \subseteq \text{info}(P) \), and it resolves the issue embodied by \( P \), i.e., \( s \in P \). But note that if \( s \in P \), then it must also be the case that \( s \subseteq \text{info}(P) \). So support just amounts to membership.

Definition 2.13 (Support)
An information state \( s \) supports a proposition \( P \) if and only if \( s \in P \).

From the fact that propositions are downward closed it follows that truth and support are closely connected.

Fact 2.14 (Truth and support)
A proposition \( P \) is true in a world \( w \) if and only if \( P \) is supported by \( \{ w \} \).

The notion of support will become very useful later on. Notice that the relation between propositions and support is exactly the same as that between issues and resolution: a proposition consists of all states that support it; an issue consists of all states that resolve it. Moreover, the relation between propositions and support in inquisitive semantics is also parallel to the relation between classical propositions and truth: a classical proposition is the set of all worlds in which it is true. In the present setting, truth does not relate directly to propositions in this way, but rather to the informative content of a proposition: the informative content of a proposition is the set of all worlds in which the proposition is true. Evidently, the fact that the connection between truth and propositions is more direct in the classical setting is an immediate consequence of the fact that classical propositions exclusively encode informative content.
2.4.2 Informative and inquisitive propositions

We will say that a proposition $P$ is informative just in case its informative content is non-trivial, i.e., $\text{info}(P) \neq W$. On the other hand, we will say that $P$ is inquisitive just in case establishing its informative content is not sufficient to settle the issue that it raises, i.e., $\text{info}(P) \notin P$.

**Definition 2.15** (Informative and inquisitive propositions)

- A proposition $P$ is informative iff $\text{info}(P) \neq W$.
- A proposition $P$ is inquisitive iff $\text{info}(P) \notin P$.

Just as we did in the case of issues, we refer to the maximal elements of a proposition as the alternatives in that proposition. These are states that support the proposition and cannot be weakened in any way without losing support. That is, they contain just enough information to support $P$.

**Definition 2.16** (Alternatives in a proposition)

- The maximal elements of a proposition $P$ are called the alternatives in $P$.
- The set of alternatives in $P$ is denoted as $\text{alt}(P)$.

When discussing issues, we noted that there is a close connection between containing multiple alternatives and being non-trivial. For propositions, there is a parallel connection between containing multiple alternatives and being inquisitive. Namely, if $P$ contains two or more alternatives, then it cannot contain $\text{info}(P)$ and therefore must be inquisitive. On the other hand, if a proposition $P$ is non-inquisitive, i.e., if $\text{info}(P) \in P$, then it always contains a unique alternative, namely $\text{info}(P)$. If a proposition contains only finitely many information states, which is the case in all the examples that we will consider, then the connection between multiple alternatives and inquisitiveness is even stronger. Namely, a proposition with finitely many elements is inquisitive if and only if it contains multiple alternatives.$^5$

**Fact 2.17** (Inquisitiveness and alternatives)

A proposition containing finitely many elements is inquisitive if and only if it contains multiple alternatives.

$^5$ See footnote 3 for an example showing that an inquisitive issue with infinitely many states does not necessarily contain multiple alternatives; it may not contain any alternatives at all. A parallel example can easily be constructed for propositions.
Figure 2.4 depicts a number of propositions. In each case, we only depict the alternatives that the proposition contains. The proposition depicted in Figure 2.4(a) contains just one alternative and is therefore not inquisitive, but it is informative, since its informative content does not cover the entire logical space. The proposition depicted in Figure 2.4(b) contains two alternatives and is therefore inquisitive; on the other hand, it is not informative, because its informative content, i.e., the union of the two alternatives, covers the entire logical space. The proposition depicted in Figure 2.4(c) is both informative and inquisitive, since it contains two alternatives and the union of these two alternatives does not cover the entire logical space. Finally, the proposition depicted in Figure 2.4(d) contains a single alternative, which covers the entire logical space; it is therefore neither informative nor inquisitive.

We will refer to a proposition that is both informative and inquisitive as a hybrid proposition, and to one that is neither informative nor inquisitive as a tautology.

Propositions can be thought of as inhabiting a two-dimensional space, as depicted in Figure 2.5. The horizontal axis is inhabited by non-inquisitive propositions, the vertical axis by non-informative propositions, the ‘zero-point’ of the space by tautologies, and the rest of the space by hybrids.
Spelling out what it means to be informative and/or inquisitive we obtain the following direct characterization of propositions that are non-informative, non-inquisitive, or tautological.

Fact 2.18

- \( P \) is non-inquisitive iff \( \text{info}(P) \in P \).
- \( P \) is non-informative iff \( \text{info}(P) = W \).
- \( P \) is a tautology iff \( W \in P \).

It will be insightful (and useful for later) to consider a number of alternative characterizations of non-inquisitive propositions as well.

Fact 2.19 (Alternative characterizations of non-inquisitive propositions)

The following are equivalent for any proposition \( P \):

1. \( P \) is non-inquisitive;
2. \( P = \wp(\text{info}(P)) \);
3. \( P \) has a greatest element;\(^6\)
4. \( P \) is supported by a state \( s \) just in case \( P \) is true in all worlds in \( s \).

Proof. We will prove the chain of implications \((1) \Rightarrow (2) \Rightarrow (3) \Rightarrow (4) \Rightarrow (1)\).

- \((1) \Rightarrow (2)\). By definition, \( \text{info}(P) = \bigcup P \), so for every \( t \in P \) we have \( t \subseteq \text{info}(P) \), which means that \( t \in \wp(\text{info}(P)) \). This shows that \( P \subseteq \wp(\text{info}(P)) \), regardless of whether \( P \) is non-inquisitive. Now suppose \( P \) is non-inquisitive, i.e., suppose \( \text{info}(P) \in P \); by downward closure, every substate of \( \text{info}(P) \) must be in \( P \) as well, so \( \wp(\text{info}(P)) \subseteq P \). Putting the two inclusions together, we obtain \( P = \wp(\text{info}(P)) \).
- \((2) \Rightarrow (3)\). If \( P = \wp(\text{info}(P)) \), clearly \( \text{info}(P) \) is the greatest element in \( P \).
- \((3) \Rightarrow (4)\). Suppose \( s \) supports \( P \), i.e., \( s \in P \). By downward closure, \( \{w\} \in P \) for all \( w \in s \). By Fact 2.14, \( P \) is true at each \( w \in s \).
Conversely, suppose \( P \) is true at each \( w \in s \). By Fact 2.14, this means that \( \{w\} \in P \) for all \( w \in s \). Suppose now that \( P \) has a greatest element, and call this element \( s^{\text{max}} \). Then for all \( w \in s \), \( \{w\} \) must be included....

\(^6\) By a greatest element we mean a state \( s^{\text{max}} \in P \) such that for every \( s \in P \), \( s \subseteq s^{\text{max}} \). Notice that if \( P \) has a greatest element \( s^{\text{max}} \), then \( s^{\text{max}} \) is the unique maximal element in \( P \). Conversely, if \( P \) has a unique maximal element, then in the finite case (but not in the general case) this is guaranteed to be the greatest element in \( P \).
in $s_{\text{max}}$, and so also $s \subseteq s_{\text{max}}$. Since $s_{\text{max}} \in P$, it follows by downward closure that $s \in P$, that is, $P$ is supported at $s$.

- $(4) \Rightarrow (1)$. Suppose $(4)$ holds. Take any $w \in \text{info}(P) = \bigcup P$: then $w \in t$ for some $t \in P$. Since $P$ is supported by $t$, by $(4)$ $P$ is true at $w$. This shows that $P$ is true at all $w \in \text{info}(P)$: by $(4)$ it follows that $P$ is supported by $\text{info}(P)$, i.e., that $\text{info}(P) \in P$. \hfill $\Box$

The characterization of non-inquisitive propositions given in $(3)$ makes it particularly easy to say whether a proposition is non-inquisitive given a visualization of it—we just have to check whether it has a greatest element. We already established in Fact 2.17 that a proposition containing finitely many elements is inquisitive if and only if it contains at least two alternatives, and thus non-inquisitive if and only if it contains just one alternative, i.e., one maximal element. The present characterization in terms of greatest elements is more general since it applies to infinite propositions as well.

The characterization of non-inquisitive propositions in $(4)$ brings out the fact that such propositions are fully characterized by their truth-conditional content. This is not the case for inquisitive propositions. For instance, the proposition depicted in Figure 2.4(b) is true at all possible worlds—it has tautological truth-conditions—yet it does not coincide with the tautological proposition in Figure 2.4(d).

Finally, one particular consequence of the characterization of non-inquisitive propositions given in $(2)$ is that there is only one proposition that counts as a tautology, namely $\wp(W)$. After all, tautologies are not only non-inquisitive but also non-informative. So if $P$ is a tautology, then we must have that $\text{info}(P) = W$. But then, according to the characterization in $(2)$, it must be the case that $P = \wp(W)$.

### 2.4.3 Entailment

Propositions can be ordered both in terms of their informative content and in terms of their inquisitive content. A proposition $P$ is at least as informative as another proposition $Q$ if and only if the informative content of $P$ determines with at least as much precision what the actual world is like as the informative content of $Q$, i.e., $\text{info}(P) \subseteq \text{info}(Q)$.

**Definition 2.20** (Informative order on propositions)

For any $P, Q \in \mathcal{P}$:

- $P \models_{\text{info}} Q$ iff $\text{info}(P) \subseteq \text{info}(Q)$
Similarly, we say that $P$ is at least as inquisitive as $Q$ just in case any state that settles the issue embodied by $P$ also settles the issue embodied by $Q$, i.e., if and only if $P \subseteq Q$.

**Definition 2.21 (Inquisitive order on propositions)**

For any $P, Q \in \mathcal{P}$:

- $P \models_{\text{inq}} Q$ if and only if $P \subseteq Q$

Combining these two orders, we say that $P$ entails $Q$ just in case $P$ is both at least as informative and at least as inquisitive as $Q$. But note that if $P \subseteq Q$, then it must also automatically hold that $\text{info}(P) \subseteq \text{info}(Q)$. So entailment simply amounts to inclusion.

**Definition 2.22 (Entailment)**

For any $P, Q \in \mathcal{P}$:

- $P \models Q$ if and only if $P \subseteq Q$

Entailment between two propositions can also be characterized as preservation of support, just like classical entailment can be characterized as preservation of truth: one proposition entails another just in case any state that supports the former also supports the latter.

**Fact 2.23 (Entailment in terms of support)**

For any $P, Q \in \mathcal{P}$:

- $P \models Q$ if and only if any state that supports $P$ also supports $Q$

Entailment forms a partial order on the set of all propositions, i.e., it is a reflexive, transitive, and anti-symmetric relation. The tautology, $\mathcal{X}(W)$, is entailed by any other proposition, i.e., it is the weakest element of the partial order. On the other hand, the partial order also has a strongest element, namely $\{\emptyset\}$, which entails all other propositions. We refer to this proposition as the contradictory proposition. We will denote the tautological and the contradictory proposition as $\top$ and $\bot$, respectively.

**Definition 2.24 (Tautology and contradiction)**

- $\top := \mathcal{X}(W)$
- $\bot := \{\emptyset\}$

**Fact 2.25 (Partial order)**

- $\models$ forms a partial order on $\mathcal{P}$
- For every $P \in \mathcal{P}$: $\bot \models P$ and $P \models \top$
2.4.4 Some linguistic examples

The notion of propositions as non-empty, downward closed sets of information states allows us to capture the informative and inquisitive content of a wide range of declarative and interrogative sentences in natural languages in a uniform and transparent way. We provide a brief illustration here; more elaborate linguistic analyses will be presented in Chapters 5–8. Imagine a context in which we are dealing with a two-digits code, where each digit can be either 1 or 0. Consider the following sentences in English.

(1)  
   a. The code is 11.
   b. The second digit of the code is 1.
   c. If the first digit is 1, the second digit is also 1.
   d. Is the code 11?
   e. What is the first digit?
   f. What is the second digit?
   g. What is the code?
   h. If the first digit is 1, what is the second?

Among these sentences, the first three are declaratives, while the remaining five are interrogatives. These sentences can all be analysed uniformly in terms of the notion of propositions developed in this section. The propositions that they express are shown in Figure 2.6, where possible worlds are identified with the corresponding codes and where, as before, only the maximal elements in each proposition—the alternatives—are displayed.

The propositions expressed by the declaratives in (1a)–(1c) each contain a single alternative, which does not cover the entire logical

**Figure 2.6** Propositions expressed by (1a)–(1h), exemplifying several types of declarative and interrogative sentences.
space. Thus, these propositions are informative but not inquisitive. This captures the fact that, in uttering one of (1a)–(1c), a speaker provides some information but does not raise any issue.

While the semantic contents of (1a)–(1c) could have been captured just as well by means of the standard notion of a proposition as a set of possible worlds, the enriched notion of propositions allows us to analyse also the interrogatives in (1d)–(1h). These are naturally taken to express the propositions depicted in Figure 2.6(d)–(h). In each case, the relevant proposition is inquisitive, since it contains multiple alternatives, and it is not informative, since these alternatives jointly cover the entire logical space. This captures the fact that, in uttering one of (1d)–(1h), a speaker raises an issue and does not provide any information.

Let us consider in some more detail the issues expressed by these questions. The polar question (1d) raises an issue which is resolved by an information state \(s\) just in case the information in \(s\) implies that the code is 11 (\(s \subseteq \{11\}\)) or it implies that the code is not 11 (\(s \subseteq \{10,01,00\}\)). The wh-question (1e) raises an issue which is resolved by an information state \(s\) just in case the information available in \(s\) determines exactly what the first digit of the code is, i.e., it implies that the first digit is 1 (\(s \subseteq \{11,10\}\)) or that the first digit is 0 (\(s \subseteq \{01,00\}\)). The situation is analogous for the question (1f). The wh-question (1g) raises an issue which is resolved by an information state \(s\) just in case the information available in \(s\) determines exactly what the code is, that is, in case \(s \subseteq \{11\}\) or \(s \subseteq \{10\}\) or \(s \subseteq \{01\}\) or \(s \subseteq \{00\}\).

Finally, the conditional wh-question (1h) raises an issue which is resolved by an information state \(s\) just in case the information available in \(s\) restricted to those worlds where the first digit is 1 determines exactly what the second digit is. This condition amounts to \(s \cap \{11,10\} \subseteq \{11\}\) or \(s \cap \{11,10\} \subseteq \{01\}\), and it is easy to see that this holds if and only if \(s \subseteq \{00,01,11\}\) or \(s \subseteq \{00,01,10\}\). Thus, the issue expressed by (1h) is one that can be settled either by establishing that if the first digit is 1, the second is also 1 (\(s \subseteq \{00,01,11\}\)) or by establishing that if the first digit is 1, the second is 0 (\(s \subseteq \{00,01,10\}\)).

The uniform perspective on the semantics of (1a)–(1h) afforded by the inquisitive notion of a proposition also gives rise to a uniform perspective on the entailment relations that hold between these sentences. As far as the declarative sentences (1a)–(1c) are concerned, we predict the same entailments that truth-conditional semantics predicts: (1a) entails (1b), which in turn entails (1c). This is as it should be: in
the absence of (proper) inquisitive content, entailment still amounts to comparing informative strength, just like in the classical case.

Now, however, the same notion of entailment can also be used to compare the questions in (1d)–(1h): in this case, informative content is trivial, and entailment will compare inquisitive strength; an entailment between a pair of questions will hold if the issue expressed by the first is at least as demanding as the issue expressed by the second. Thus, for instance, (1g) entails any of the other questions: if one establishes what the code is, one also thereby establishes whether the code is 11, what the first digit is, and so on. More generally, the issue expressed by (1g) is the strongest possible issue over the ignorant state.

The analysis also captures that (1f) entails (1h): if one establishes what the second digit is, one also thereby establishes what the second digit is if the first digit is 1. Another prediction is that the questions in (1d) and (1e) are incomparable in terms of inquisitive strength: on the one hand, the information that the code is not 11 resolves (1d) but not (1e); on the other hand, the information that the first digit is 1 resolves (1e) but not (1d); thus, neither of these questions entails the other.

Finally, notice that entailment can also be used to compare declaratives with interrogatives. A declarative entails an interrogative just in case the information provided by the former suffices to resolve the issue raised by the latter. Thus, (1a), which completely specifies the code, entails all the questions in (1d)–(1h); (1b), which only specifies the second digit, entails only (1f) and (1h); and (1c), which gives conditional information about the second digit, entails only (1h). Conversely none of the above questions entails any of the given declaratives. This is because, in terms of informative content, (1d)–(1h) are trivial, while (1a)–(1c) are not.

2.5 Contexts

In Section 1.1.1 we reviewed a number of reasons why conversational contexts should be modeled in a way that does not only take account of the information that has been established in the conversation so far, but also of the issues that have been brought up, often referred to as questions under discussion. This can be done using the notion of issues introduced above. The most straightforward way of doing so would

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7 It can also be done—and indeed has been done—using different formal notions from the literature on questions (e.g., Hamblin, 1973; Groenendijk and Stokhof, 1984;
be to model a context $C$ as a pair $\langle \text{info}_C, \text{issues}_C \rangle$, where $\text{info}_C$ is an information state representing the information that the conversational participants have commonly established so far, and $\text{issues}_C$ a set of issues which have been raised in the conversation so far and which the conversational participants would like to see commonly resolved. That is, while $\text{info}_C$ represents the current common ground, the issues in $\text{issues}_C$ determine what kind of common ground the conversational participants would like to establish, namely one in which every issue in $\text{issues}_C$ is settled. The initial context would then be $\langle W, \emptyset \rangle$, consisting of the trivial information state, which does not rule out any world, and the empty set of issues. As the conversation progresses, worlds would be removed from $\text{info}_C$ and issues would be added to $\text{issues}_C$.

This way of modeling conversational contexts is a good starting point, but just as in the case of propositions, it is natural to impose certain constraints on how the informative and the inquisitive component of a context are related to each other. First, every issue $I \in \text{issues}_C$ should be one that is settled only in information states that enhance the current common ground, $\text{info}_C$. Formally, this means that for every $I \in \text{issues}_C$ we should have that $\bigcup I \subseteq \text{info}_C$.8

Second, for every issue $I \in \text{issues}_C$, the information available in the current common ground should ensure that $I$ can be resolved truthfully, i.e., without discarding the actual world. This means that for every $I \in \text{issues}_C$ we should have that $\text{info}_C \subseteq \bigcup I$. To see this, we can follow the same line of reasoning that we followed when considering propositions above. Thus, for every $I \in \text{issues}_C$, it should hold on the one hand that $\bigcup I \subseteq \text{info}_C$ and on the other hand that $\text{info}_C \subseteq \bigcup I$. Putting the two together, we get that every $I \in \text{issues}_C$ should be an issue over $\text{info}_C$: $\bigcup I = \text{info}_C$.

If we impose this constraint, the considered notion of conversational contexts is in principle a suitable notion. However, for our current purposes, it can be simplified. We will do this in two steps. First, rather than thinking of a context $C$ as a pair $\langle \text{info}_C, \text{issues}_C \rangle$ where $\text{issues}_C$ is a set of issues over $\text{info}_C$, we may just as well think of it

---

8 In a conversation, it is of course possible to raise issues which call into question some of the propositions which were part of the common ground. However, in order to countenance such an issue, the common ground first needs to be weakened so as to make the relevant issue again open for debate. We refer to Ciardelli and Roelofsen (2014) for some discussion of how the process of dropping a certain belief could be modeled in the inquisitive setting.
as a pair \((\text{info}_C, \text{issue}_C)\) where \(\text{issue}_C\) is a single issue over \(\text{info}_C\). This simplification is justified by the fact that any set of issues \(\Omega\) over a state \(s\) can be merged into a single issue over \(s\):

\[
I_\Omega := \{ t \subseteq s \mid t \in J \text{ for every } J \in \Omega \}
\]

which is settled precisely by those enhancements \(t \subseteq s\) that settle all issues in \(\Omega\). Notice that if \(\Omega \neq \emptyset\) the issue \(I_\Omega\) amounts to the intersection \(\bigcap \Omega\) of all issues in \(\Omega\), whereas if \(\Omega = \emptyset\), \(I_\Omega\) amounts to the trivial issue \(\wp(s)\) over \(s\).

So we can think of a context \(C\) as a pair \((\text{info}_C, \text{issue}_C)\), where \(\text{info}_C\) is an information state and \(\text{issue}_C\) a single issue over \(\text{info}_C\). We can then take the initial context to be the pair \((W, \wp(W))\), consisting of the trivial information state, which does not rule out any world, and the trivial issue over this state, which is settled even if no information is present yet.

But this representation can be simplified further. After all, since \(\text{issue}_C\) is an issue over \(\text{info}_C\), we always have that \(\text{info}_C = \bigcup \text{issue}_C\). That is, \(\text{info}_C\) can always be retrieved from \(\text{issue}_C\). But then \(\text{info}_C\) can just as well be left out of the representation of \(C\). Thus, a context \(C\) can simply be represented as an issue, i.e., a non-empty, downward closed set of information states. The information commonly established in \(C\) is then embodied by \(\bigcup C\).

**Definition 2.26 (Contexts)**

- A context \(C\) is a non-empty, downward closed set of information states.
- The set of all contexts will be denoted by \(\mathcal{C}\).

**Definition 2.27 (The information available in a context)**

- For any context \(C\): \(\text{info}(C) := \bigcup C\)

---

\(^9\) Recall from footnote 1 that we are implicitly already assuming a similar simplification concerning the informative component of a context: we do not keep track of all the separate pieces of information that have been established in the conversation so far, but rather of the set of worlds that are compatible with all these pieces of information—formally, this is again the intersection of all the separately established pieces of information. For certain purposes it is necessary to keep track of all the separate pieces of information and/or issues that have been established/raised in a conversation (see, e.g., Roberts, 1996; Farkas and Bruce, 2010; Farkas and Roelofsen, 2017). For our current purposes, however, this would only add unnecessary complexity.

\(^{10}\) Notice that \(I_\Omega\) is guaranteed to be an issue in the sense of Definition 2.3. In particular, it is guaranteed to be non-empty, since it always contains the inconsistent information state.
We have moved from the commonplace notion of a context as a set of possible worlds—representing the information established so far—to a richer notion of contexts as non-empty, downward closed sets of information states—representing both the information established and the issues raised so far. We will now identify some special properties that contexts may have (§2.5.1), some relations that may hold between them (§2.5.2), and some operations that can be performed on them (§2.5.3).

### 2.5.1 Informed and inquisitive contexts

First of all, we say that a context $C$ is **informed** just in case some non-trivial information has been established in it, i.e., $\text{info}(C) \neq W$. Otherwise we say that the context is **uninformed**.

**Definition 2.28** (Informed and uninformed contexts)

- A context $c$ is informed iff $\text{info}(C) \neq W$.
- A context $c$ is uninformed iff $\text{info}(C) = W$.

Similarly, we say that a context $C$ is **inquisitive** just in case the information that has been established so far does not yet settle the issues that have been raised, i.e., $\text{info}(C) \notin C$. On the other hand, if all issues are settled we say that $C$ is **indifferent**.

**Definition 2.29** (Inquisitive and indifferent contexts)

- A context $C$ is inquisitive iff $\text{info}(C) \notin C$.
- A context $C$ is indifferent iff $\text{info}(C) \in C$.

There are two special contexts: the **initial** and the **absurd** context. The initial context, $C_\top$, is the only context that is both uninformed and indifferent. The absurd context, $C_\bot$, is the one in which the established information is inconsistent and therefore rules out all possible worlds.

**Definition 2.30** (The initial and the absurd context)

- $C_\top := \wp(W)$
- $C_\bot := \{\emptyset\}$

Some example contexts are depicted in Figure 2.7, whereas before it is assumed that $W = \{w_1, w_2, w_3, w_4\}$. Only the maximal states in each context are depicted. Since contexts are downward closed, we know that all enhancements of these maximal states are also part of the context at hand. The context in (a) is the initial context, $\wp(W)$, which is neither
informed nor inquisitive. The one in (b) is still not informed, but it is inquisitive. In order to resolve the issue that is present in this context, it either needs to be established that the actual world is one of \{w_1, w_2\} or that it is one of \{w_3, w_4\}. The context in (c) is both informed and inquisitive. In this context it is common ground that the actual world is one of \{w_1, w_2, w_3\}, i.e., \(w_4\) has been ruled out as a candidate for the actual world, but in order to resolve the issue that has been raised, more precise information is needed—namely, it either needs to be established that the actual world is one of \{w_1, w_2\} or that it is one of \{w_1, w_3\}. Finally, the context in (d) is informed, but not inquisitive. It is common ground in this context that the actual world is one among \{w_1, w_2\}, and no issues have been raised whose resolution would require more precise information.

2.5.2 Context extension

Two contexts can be compared in terms of the information that has been established or in terms of the issues that have been raised. One context \(C'\) is at least as informed as another context \(C\) if and only if \(\text{info}(C') \subseteq \text{info}(C)\).

**Definition 2.3.1 (Informative order on contexts)**

For any contexts \(C, C'\):

- \(C' \succeq_{\text{info}} C\) iff \(\text{info}(C') \subseteq \text{info}(C)\)

Similarly, we say that \(C'\) is at least as inquisitive as \(C\) if and only if every state that settles all the issues that have been raised in \(C'\) also settles all the issues that have been raised in \(C\), i.e., if and only if \(C' \subseteq C\).

**Definition 2.3.2 (Inquisitive order on contexts)**

For any contexts \(C, C'\):

- \(C' \succeq_{\text{inq}} C\) iff \(C' \subseteq C\)
Combining these two orders, we say that $C'$ is an extension of $C$ just in case $C'$ is both at least as informed and at least as inquisitive as $C$. But note that if $C' \subseteq C$, then it must also be the case that $\text{info}(C') \subseteq \text{info}(C)$. So context extension simply amounts to inclusion.

**Definition 2.33 (Extending contexts)**
For any contexts $C, \ C'$:

- $C'$ is an extension of $C$, $C' \geq C$, iff $C' \subseteq C$

The extension relation forms a *partial order* on $C$, and $C\top$ and $C\bot$ constitute the extremal elements of this partial order: $C\bot$ is an extension of every context, and every context is in turn an extension of $C\top$.

**Fact 2.34 (Partial order)**

- $\geq$ forms a partial order on $C$
- For every $C \in C$: $C\bot \geq C$ and $C \geq C\top$

In Figure 2.7, the contexts in (b), (c), and (d) are all extensions of the trivial context in (a). Moreover, (d) is also an extension of (b) and (c), but neither (b) nor (c) is an extension of the other.

### 2.5.3 Updating contexts

Recall that in the standard setting, where both contexts and propositions are construed as sets of possible worlds, the result of updating a context $c$ with a proposition $p$ is a new context $c[p]$ which, besides the information already present in $c$, also contains the information embodied by $p$. That is, a candidate for the actual world is ruled out by $c[p]$ if it was already ruled out by the information established in the old context $c$, or if it is ruled out by the new information embodied by $p$. Thus, formally, update amounts to *set intersection* in the standard setting: $c[p] = c \cap p$.

In inquisitive semantics, we want the result of updating a context $C$ with a proposition $P$ to be a new context $C[P]$ which incorporates both the informative content of $P$ and the issue that it embodies. Thus, on the one hand, a candidate for the actual world must be ruled out by the information established in $C[P]$ if it was either already ruled out by the information established in the old context $C$, or if it is ruled out by the informative content of $P$. Formally, this means that we must have that $\text{info}(C[P]) = \text{info}(C) \cap \text{info}(P)$. On the other hand, a state must resolve the issues present in $C[P]$ if and only if it resolves the
issues already present in $C$ and also the issue embodied by $P$. Formally, this means that we must have that $C[P] = C \cap P$. Now, note that if the latter condition is satisfied, then the former condition is automatically satisfied as well. This means that, just as in the standard setting, update can simply be defined as set intersection.

**Definition 2.35 (Updating contexts)**
For any $C \in \mathcal{C}$ and any $P \in \mathcal{P}$:

- $C[P] := C \cap P$

Some examples of context update are given in Figure 2.8. In the first case, the initial context is informed but not inquisitive. More specifically, in this context it is commonly established that the actual world is one among $\{w_1, w_2, w_3\}$, and no issues have been raised that require more precise information. This context is updated with a proposition which is informative—embodying the information that the actual world is one among $\{w_1, w_2, w_4\}$—but not inquisitive. The result of the update, obtained by intersection, is a new context in which it is established that the actual world is among $\{w_1, w_2\}$, and where there are still no issues.

<table>
<thead>
<tr>
<th>Initial context</th>
<th>Proposition</th>
<th>New context</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w_1$ $w_2$ $w_3$ $w_4$</td>
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<td>$w_1$ $w_2$ $w_3$ $w_4$</td>
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<td>$w_1$ $w_2$ $w_3$ $w_4$</td>
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</table>

*Figure 2.8 Some update examples.*
that require more precise information. Note that in this case, where neither the initial context nor the proposition involved in the update are inquisitive, our framework reproduces exactly the same result that is obtained in the standard setting. This holds in full generality.

Fact 2.36 (Update without inquisitiveness yields standard results)
For any non-inquisitive context $C$ and any non-inquisitive proposition $P$, $C[P]$ is a non-inquisitive context as well, and its unique maximal element is the intersection of the unique maximal element of $C$ and that of $P$.

The second example in Figure 2.8 is one where the initial context is the same as in the first example, but now the proposition with which it is updated is inquisitive, embodying the issue of whether the actual world is among $\{w_1, w_2\}$ or among $\{w_3, w_4\}$. The context resulting from the update is one in which this issue is present, together with the information that was already available beforehand. That is, after the update it is still established that the actual world is among $\{w_1, w_2, w_3\}$, as in the initial context, but now there is also an issue as to whether it is $w_3$ or among $\{w_1, w_2\}$. Note that in order to obtain this result simply by means of intersection, it is important that both contexts and propositions are downward closed. Made fully explicit, the initial context is represented as the following set of information states:

$\{\{w_1, w_2, w_3\}, \{w_1, w_2\}, \{w_1, w_3\}, \{w_2, w_3\}, \{w_1\}, \{w_2\}, \{w_3\}, \emptyset\}$

The proposition considered is:

$\{\{w_1, w_2\}, \{w_3, w_4\}, \{w_1\}, \{w_2\}, \{w_3\}, \{w_4\}, \emptyset\}$

Applying intersection to these two sets yields the new context:

$\{\{w_1, w_2\}, \{w_1\}, \{w_2\}, \{w_3\}, \emptyset\}$

whose two maximal elements, $\{w_1, w_2\}$ and $\{w_3\}$, are the ones that are depicted. This result would not be obtained if we discharged downward closure and identified contexts and propositions exclusively with their maximal elements. In that case, the initial context would be represented as $\{\{w_1, w_2, w_3\}\}$, the proposition at hand as $\{\{w_1, w_2\}, \{w_3, w_4\}\}$, and applying intersection to these two sets would yield the empty set, clearly not the desired result.

The third example in Figure 2.8 is one where the initial context is already inquisitive. The issue that is present is whether the actual world is among $\{w_1, w_3\}$ or among $\{w_2, w_4\}$. The proposition with which this
The update results in a context in which these two issues have been merged. In order to resolve the issue that is present in this new context it is necessary to determine exactly which of \( w_1, w_2, w_3, w_4 \) is the actual world. That is, it is necessary to resolve the issue that was already present in the initial context, and also the issue that was embodied by the proposition involved in the update.

Thus, while our update procedure yields standard results in the case of non-inquisitive contexts and propositions, it also smoothly generalizes to cases involving inquisitive contexts and/or propositions.

2.6 Summary and pointers to possible refinements

We have now introduced all the notions that we set out to introduce (recall the diagram in Figure 2.1 at the beginning of the chapter). We adopted the standard notion of information states as sets of possible worlds. In terms of this familiar notion, we defined a new notion of issues. We represent an issue as a non-empty, downward closed set of information states, namely those information states that contain enough information to resolve the issue. With this crucial notion in place, we turned to propositions and contexts. We moved from the standard notion of a proposition as a set of possible worlds, which just allows us to capture the information that a sentence conveys, to a more fine-grained notion, which also allows us to capture the issue that a sentence raises. Similarly, we replaced the standard minimal notion of contexts, which just captures the information that has been commonly established in the conversation so far, by a richer notion that also allows us to capture the issues that have been brought up. Formally, both propositions and contexts are not modeled as sets of possible worlds in our framework, but rather, just like issues, as non-empty, downward closed sets of information states.

Turning to the relations that may hold between the various kinds of objects, we have seen that entailment between propositions, enhancement of information states, and extension of contexts all amount to set inclusion, just as in the standard setting, and the same is true for the new notion of issue refinement. Support, a relation between information states and propositions, is no longer defined as inclusion, but rather as membership. This is a consequence of the fact that an information state
no longer necessarily supports a proposition if it implies the informative content of that proposition; rather, it should also contain enough information to resolve the issue embodied by the proposition. Finally, context update still amounts to set intersection. However, since the operation no longer applies to sets of worlds but rather to sets of information states, we have seen that it can deal in a uniform way with cases involving purely informative propositions and indifferent contexts, and with cases involving inquisitive propositions and/or contexts.

We briefly illustrated how the informative and inquisitive content of various types of sentences in English can be captured using the proposed notion of propositions. There are also several aspects of meaning that are beyond the scope of the basic inquisitive semantics framework that we are presenting here. However, the framework is set up in such a way that it allows for several natural refinements. We briefly mention four such refinements, with references to other work for further detail.

First, instead of the static view on meaning that we have assumed here, one may also adopt a dynamic view on meaning (see, e.g., Kamp, 1981; Heim, 1982; Groenendijk and Stokhof, 1991; Veltman, 1996). Under this view, the meaning of a sentence is conceived of as its context change potential, modeled formally as a function $F$ that maps any context $C$ to a new context $F(C)$, which would result from uttering the given sentence in $C$. This new context $F(C)$ need not necessarily be obtained by intersecting $C$ with the proposition $P$ expressed by the sentence. In fact, on a dynamic view, there is no need to associate sentences with propositions at all. This allows for greater flexibility, which has led to important advances in the treatment of various linguistic phenomena, including anaphora and presuppositions. While we have taken a static perspective here, the formal notions that we have introduced, in particular the notion of context, also form a suitable starting point for a dynamic inquisitive semantics (see Ciardelli et al., 2012, 2013a, for initial work in this direction, though much remains to be done).

Second, rather than starting out with the commonplace notion of information states as sets of possible worlds, which imposes a very specific (Boolean) structure on the space of information states, we may also work with other notions of information states, giving rise to information spaces with different structures. This strategy can be used, in particular, to implement inquisitive semantics in a context where the underlying view of information is non-classical (Punčochář, 2017; Ciardelli et al., 2017b).
Third, in order to model more than just informative and inquisitive content we may further enrich our notion of propositions and/or contexts, either by explicitly encoding additional dimensions of meaning (see, e.g., Roelofsen and Farkas, 2015; AnderBois, 2016b), or by weakening the downward closure constraint that we have placed on contexts and propositions here (Ciardelli et al., 2014; Pučočhář, 2015; Groenendijk and Roelofsen, 2015). Such amendments lead to richer notions of meaning, and further broaden the range of linguistic phenomena that can be captured in the framework. However, these refinements also involve certain complications that do not arise in the basic framework presented here.

Finally, in addition to the basic, incremental notion of context update discussed in this chapter—where contexts are always enhanced by adding more information or more issues—one might consider more complex and realistic models of conversation, allowing conversational participants to reject a given proposal or resist it in less drastic ways (Bledin and Rawlins, 2016), as well as to retract previous claims and challenge some of the information in the common ground.

2.7 Exercises

Exercise 2.1 Contexts

1. Give a representation of the following contexts:
   (a) it is established that Bill is going to the party, and there is an issue as to whether Mary is going as well;
   (b) it is established that if Bill goes to the party, then Mary will go as well, and there is an issue as to whether Bill is going;
   (c) it is established that only one of Bill and Mary is going to the party, and there is an issue as to which of them is going.

2. For each of these contexts, determine whether it is an extension of the others.

Exercise 2.2 Propositions

1. Give a representation of the propositions encoding the following information and issues:
   (a) Information: that Bill is only going to the party if Mary is going. Issue: whether Mary is going.
2.7 EXERCISES

(b) Information: none.
Issue: which among Bill and Mary are going to the party (only Bill, only Mary, both, or neither).

(c) Information: that only Mary is going to the party, not Bill.
Issue: none.

2. For each of the above propositions, determine whether it entails the others.

Exercise 2.3 Update

1. Determine the result of updating each of the contexts in Exercise 2.1 with each of the propositions in Exercise 2.2.

2. Prove Fact 2.36, showing that the notion of update as intersection yields standard results when applied to non-inquisitive contexts and propositions.

Exercise 2.4 Informational and inquisitive triviality

Let $P$ and $P'$ be two non-inquisitive propositions, and $Q$ and $Q'$ two non-informative propositions.

1. Is $P \cap P'$ guaranteed to be non-inquisitive? If so, give a proof; if not, give a counterexample.

2. Is $Q \cap Q'$ guaranteed to be non-informative? If so, give a proof; if not, give a counterexample.

3. Is $P \cap Q$ either guaranteed to be non-inquisitive or to be non-informative? If so, give a proof; if not, give a counterexample.
Now that we have introduced a new notion of propositions, it is natural to consider what the basic operations are that could be performed on such propositions. In the classical setting, where propositions are simple sets of worlds, we can form the intersection or the union of two propositions, or the complement of a single proposition. These operations play a central role in logic and in semantic analyses of natural languages: conjunction and disjunction are standardly taken to express intersection and union, respectively, while negation is standardly taken to express complementation. Do these operations have natural counterparts in the inquisitive setting, where propositions are no longer simple sets of worlds?

We will address this question in Section 3.1, adopting an algebraic perspective. We will find that the basic algebraic operations on classical propositions can indeed be applied to inquisitive propositions as well. This result facilitates a very natural way of dealing with connectives and quantifiers. In particular, in Chapter 4 it will allow us to define an inquisitive semantics for the language of first-order logic which is, from an algebraic perspective, the exact counterpart of classical first-order logic in the inquisitive setting. In Chapter 5 we will suggest that the algebraic operations are also crucial for the semantic analysis of connectives in natural languages. In particular, they will yield a uniform account of conjunction, disjunction, and conditionalization of statements and questions, as illustrated by examples (1)–(3) below, repeated from Section 1.1.3.

(1)  a. Peter rented a car and Mary booked a hotel.
    b. Where can we rent a car, and which hotel should we take?

(2)  a. Peter rented a car or he borrowed one.
    b. Where can we rent a car, or who has one that we could borrow?
Returning now to the roadmap for the present chapter, after having discussed the basic algebraic operations on inquisitive propositions in Section 3.1, we will consider two other natural operations in Section 3.2, namely ones that trivialize the informative or the inquisitive content, respectively, of any given proposition. For reasons that will become clear below, we refer to such operators as projection operators. One projection operator turns any proposition into a corresponding non-inquisitive proposition, while the other turns any proposition into a corresponding non-informative proposition. Clearly, these operations do not have a counterpart in the classical setting, where propositions capture only informative content to begin with; but in the inquisitive setting they naturally arise, and we will suggest that they also have an important role to play in the semantic analysis of natural languages. More specifically, in Chapter 6 we will use these projection operators to capture the semantic contribution of declarative and interrogative complementizers.

3.1 Algebraic operations

In this section we will identify the basic algebraic operations that can be applied to inquisitive propositions. To illustrate our approach, we will first briefly review the algebraic perspective on classical logic.

3.1.1 The algebraic perspective on classical logic

In the classical setting a proposition $P$ is simply a set of possible worlds. Let us denote the set of all classical propositions as $\mathcal{P}_c$. The proposition expressed by a sentence can be thought of as carving out a certain region in the logical space—the set of all possible worlds—and in asserting a sentence, a speaker is taken to provide the information that the actual world is located within this region. One proposition $P$ entails another proposition $Q$, $P \models Q$, just in case $P \subseteq Q$, which means that $P$ carves out a smaller region in the logical space than $Q$ does, thereby encoding more information as to what the actual world is like. Entailment forms a partial order on the set of all classical propositions, i.e., it is a reflexive, transitive, and anti-symmetric relation. In Figure 3.1 we have depicted the set of all classical propositions, assuming that the logical space
Figure 3.1 The set of all propositions in classical semantics if the logical space consists of two possible worlds (on the left) or three possible worlds (on the right). Arrows indicate entailment.

consists of two worlds (the diagram on the left) or of three worlds (the diagram on the right). In each case, the arrows indicate entailment.

Now, given a partially ordered set, we can ask what natural operations can be defined on it—that is, what algebraic structure it has. The set of classical propositions ordered by entailment, \( \langle \mathcal{P}_{cl}, \models \rangle \), forms a so-called Heyting algebra, which comes with four basic operations: meet, join, relative pseudo-complementation and absolute pseudo-complementation.\(^1\)

To illustrate these operations, we will make use of the diagrams in Figure 3.2. The grid in each diagram represents a space of propositions ordered by entailment: each node in the grid is a proposition, and one proposition entails another just in case there is a path from the first to the second along the grid such that each step of the path goes up. For instance, in Figure 3.2(a) \( P \) entails \( Q \), because there is a path going up from \( P \) to \( Q \), but \( P \) does not entail \( R \) because any path from \( P \) to \( R \) must contain at least one step that goes down.

\(^1\) In fact, as we will discuss, the space of classical propositions ordered by entailment is a special kind of Heyting algebra, namely a Boolean algebra. For us, however, the more general fact that it forms a Heyting algebra will be crucial, because we will find that in inquisitive semantics the space of propositions no longer forms a Boolean algebra, but does still form a Heyting algebra. This will allow us to construe the basic logical operators in inquisitive semantics as the exact counterparts of those in classical logic.
The *meet* of $P$ and $Q$ is the greatest lower bound of $P$ and $Q$ with respect to entailment, i.e., the weakest proposition that entails both $P$ and $Q$. As indicated in Figure 3.2(b), this greatest lower bound amounts to the intersection of the two propositions: $P \cap Q$. More generally, the meet of a (possibly infinite) set of propositions $\Sigma$ amounts to the intersection of all the propositions in that set:

$$\bigcap \Sigma = \{ w \in W : w \in P \text{ for all } P \in \Sigma \}$$

If $\Sigma$ is empty, then $\bigcap \Sigma$ is the proposition consisting of all possible worlds, $W$. This is the weakest of all propositions, since it is entailed by all other propositions. It is denoted as $\bot$. On the other hand, if $\Sigma$ is the set of all propositions, then $\bigcap \Sigma$ is the empty proposition, $\emptyset$. This is the strongest of all propositions, since it entails all other propositions. It is denoted as $\top$.

The *join* of two propositions $P$ and $Q$ is the least upper bound of $P$ and $Q$ with respect to entailment, i.e., the strongest proposition that is entailed by both $P$ and $Q$. As indicated in Figure 3.2(b), this least upper bound amounts to the union of the two propositions: $P \cup Q$. More generally, the join of a (possibly infinite) set of propositions $\Sigma$ amounts to the union of all the propositions in that set:

$$\bigcup \Sigma = \{ w \in W : w \in P \text{ for some } P \in \Sigma \}$$

If $\Sigma$ is empty, then $\bigcup \Sigma$ is the empty proposition, $\bot$. On the other hand, if $\Sigma$ is the set of all propositions, then $\bigcup \Sigma$ is the proposition consisting of all possible worlds, $\top$.

The existence of meets and joins for arbitrary sets of propositions implies that $\langle \mathcal{P}_d, \models \rangle$ forms a complete lattice, bounded by $\bot$ and $\top$ as its strongest and weakest elements, respectively.

Now let us turn to relative and absolute pseudo-complementation. The pseudo-complement of a proposition $P$ relative to another
proposition $Q$, which we will denote as $P \Rightarrow Q$, can be thought of intuitively as the *difference* between $P$ and $Q$: it is the weakest proposition $R$ such that $P$ and $R$ together contain at least as much information as $Q$. More formally, it is the weakest proposition $R$ such that $P \bigcap R \models Q$. To illustrate this notion, consider the propositions $P$ and $Q$ in Figure 3.2(c). How do we find the pseudo-complement of $P$ relative to $Q$? The first step is to determine the set of all propositions $R$ which are such that $P \bigcap R \models Q$. This is the shaded area in Figure 3.2(c). Indeed, if we take the meet of $P$ with any proposition in this area, we obtain a proposition that entails $Q$. The second step is to select the weakest element of this set, i.e., that proposition which is entailed by all other propositions in the set. In our diagrams, one proposition entails another if there is a path going up from the first to the second. This means that the weakest element of a set of propositions is the topmost one. Thus, the topmost element of the shaded area in Figure 3.2(c) is the pseudo-complement of $P$ relative to $Q$, which is denoted as $P \Rightarrow Q$.

It can be shown that $P \Rightarrow Q$ always consists of all possible worlds which, if contained in $P$, are also contained in $Q$:

$$P \Rightarrow Q = \{ w \mid \text{if } w \in P \text{ then } w \in Q \text{ as well} \}$$

Absolute pseudo-complementation is a limit case of its relative counterpart. The absolute pseudo-complement of a proposition $P$, which we will denote as $P^*$, is the weakest proposition $R$ such that $P$ and $R$ are *incompatible*, in the sense that $P$ and $R$ together yield a contradiction. More formally, $P^*$ is the weakest proposition $R$ such that $P \bigcap R = \bot$. This means that $P^*$ amounts to $P \Rightarrow \bot$, the pseudo-complement of $P$ relative to $\bot$. Within the space of classical propositions, the pseudo-complement of $P$ is simply the set-theoretic complement of $P$:

$$P^* = \{ w \mid w \not\in P \}$$

In a Heyting algebra it always holds, by definition of $P^*$, that $P \bigcap P^* = \bot$. In the specific case of $\langle P_{cl}, \models \rangle$, we also always have that $P \bigcup P^* = \top$. This means that in this particular setting, $P^*$ is in fact the *Boolean complement of $P$, and that $\langle P_{cl}, \models \rangle$ forms a Boolean algebra, a special kind of Heyting algebra.

Thus, classical propositions are amenable to certain basic algebraic operations. Classical first-order logic is obtained by associating these operations with the connectives and the quantifiers. Indeed, the usual definition of truth can be reformulated as a recursive definition of the
basic operations on propositions

set $|\varphi|$ of worlds in which $\varphi$ is true (given a domain $D$ of individuals).
The inductive clauses then run as follows:

- $\neg \varphi = \varphi^*$
- $\varphi \land \psi = \varphi \cap \psi$
- $\varphi \lor \psi = \varphi \cup \psi$
- $\varphi \rightarrow \psi = \varphi \Rightarrow \psi$
- $\forall x. \varphi(x) = \bigcap_{d \in D} \varphi(d)$
- $\exists x. \varphi(x) = \bigcup_{d \in D} \varphi(d)$

Negation expresses absolute pseudo-complementation; conjunction and disjunction express binary meet and join, respectively; implication expresses relative pseudo-complementation; and quantified formulas, $\forall x. \varphi$ and $\exists x. \varphi$, express the infinitary meet and join, respectively, of $\{|\varphi(d)| \mid d \in D\}$.

Notice that everything started with a notion of propositions and a natural entailment order on these propositions. The entailment order induces certain basic operations on propositions, and classical first-order logic is obtained by associating these basic semantic operations with the connectives and quantifiers.

3.1.2 Algebraic operations on inquisitive propositions

Recall that in inquisitive semantics propositions are not sets of worlds, but rather sets of information states, non-empty and downward closed. In this setting, one proposition $P$ entails another proposition $Q$ just in case $P$ is at least as informative and at least as inquisitive as $Q$. We have seen that this condition is satisfied just in case $P \subseteq Q$. So technically entailment still amounts to inclusion, just like in classical logic, though now it encompasses both informative and inquisitive strength. In Figure 3.3 we have depicted the set of all propositions in a logical space consisting of two possible worlds, and in Figure 3.4 we have done the same for a logical space consisting of three possible worlds. As before, arrows indicate entailment.

Now let us consider the algebraic structure of the space of all inquisitive propositions ordered by entailment, $\langle P, \models \rangle$, in order to determine which operations could be associated with the connectives and the quantifiers in an inquisitive semantics for the language of first-order logic. What kind of algebraic operations can be performed on inquisitive propositions? Does every set of propositions still have a unique greatest
Figure 3.3: The set of all inquisitive semantics propositions if the logical space consists of two possible worlds. Arrows indicate entailment.

lower bound (meet) and a unique least upper bound (join) with regard to entailment? Does every proposition still have a pseudo-complement relative to any other proposition?

It turns out that these questions can be answered in the positive: $(\mathcal{P}, \models)$ forms a complete Heyting algebra, just like $(\mathcal{P}_d, \models)$. First, any set of propositions $\Sigma \subseteq \mathcal{P}$ still has a meet and a join, which can moreover still be characterized in terms of intersection and union.

Fact 3.1 (Meet)
Any set of propositions $\Sigma \subseteq \mathcal{P}$ has a meet, which amounts to:

$$\bigwedge \Sigma = \{ s \mid s \in P \text{ for all } P \in \Sigma \}$$

Fact 3.2 (Join)
Any set of propositions $\Sigma \subseteq \mathcal{P}$ has a join, which amounts to:

$$\bigvee \Sigma = \{ s \mid s \in P \text{ for some } P \in \Sigma \}$$

if $\Sigma \neq \emptyset$, and to $\{\emptyset\}$ otherwise.

The existence of meets and joins for arbitrary sets of propositions implies that $(\mathcal{P}, \subseteq)$ forms a complete lattice. This lattice has a unique strongest element, $\bot := \{\emptyset\}$, and a unique weakest element, $\top := \wp(W)$.

Furthermore, just as in the classical setting, for every two propositions $P$ and $Q$, there is a unique weakest proposition $R$ such that $P \cap R$
Figure 3.4 The set of all inquisitive semantics propositions if the logical space consists of three possible worlds. Arrows indicate entailment.
3.1 Algebraic Operations

entails $Q$. Recall that this proposition, the pseudo-complement of $P$ relative to $Q$, can be thought of intuitively as the difference between $P$ and $Q$.

**Fact 3.3** (Relative pseudo-complement)
For any $P, Q \in \mathcal{P}$, the pseudo-complement of $P$ relative to $Q$ amounts to:

$$P \Rightarrow Q := \{s \mid \text{for every } t \subseteq s, \text{if } t \in P \text{ then } t \in Q\}$$

The existence of relative pseudo-complements implies that $(\mathcal{P}, \subseteq)$ forms a Heyting algebra. Finally, recall that the absolute pseudo-complement of a proposition $P$, denoted $P^*$, is defined as the pseudo-complement of $P$ relative to $\bot$. We saw that in the classical setting, $P^*$ amounts to the set of worlds that are not in $P$. In the inquisitive setting, $P^*$ amounts to the set of states that are incompatible with any state in $P$.

**Fact 3.4** (Absolute pseudo-complement)
For any proposition $P \in \mathcal{P}$:

$$P^* = \{s \mid s \cap t = \emptyset \text{ for all } t \in P\}$$

A state $s$ is incompatible with all states in $P$ just in case it is incompatible with the union $\bigcup P$ of all these states, which amounts to $\text{info}(P)$. In turn, $s$ is incompatible with $\text{info}(P)$ just in case $s \subseteq \overline{\text{info}(P)}$. This leads to the following alternative characterization of $P^*$.

**Fact 3.5** (Absolute pseudo-complements, alternative characterization)
For any proposition $P \in \mathcal{P}$:

$$P^* = \overline{\phi(\text{info}(P))}$$

This characterization shows in particular that the absolute pseudo-complement of any given proposition $P$ always contains a single alternative, $\overline{\text{info}(P)}$, and is therefore never inquisitive.

The algebraic operations that we have identified are exactly the ones that are present in the classical setting. One notable difference, however, is that the absolute pseudo-complement of an inquisitive proposition is not always its Boolean complement. In fact, most inquisitive propositions do not have a Boolean complement at all. To see this, suppose that $P$ and $Q$ are Boolean complements. This means that:

(i) $P \cap Q = \bot$
(ii) $P \cup Q = \top$
Since $\top = \wp(W)$, condition (ii) can only be fulfilled if either $P$ or $Q$ contains $W$. Suppose $W \in P$. Then, since $P$ is downward closed, $P = \wp(W) = \top$. But then, in order to satisfy condition (i), we must have that $Q = \{\emptyset\} = \bot$. So the only two elements of our algebra that have a Boolean complement are $\top$ and $\bot$. Hence, the space $\langle P, \models \rangle$ of inquisitive propositions does not form a Boolean algebra, unlike the space $\langle P_{\text{cl}}, \models \rangle$ of classical propositions.

This difference has repercussions for the behavior of the logical system that we will specify, in particular for negation (for instance, the law of double negation will no longer hold). However, the similarity between $\langle P, \models \rangle$ and $\langle P_{\text{cl}}, \models \rangle$ that we identified, i.e., the fact that both form a Heyting algebra, is much more important for our current purposes. In particular, the existence of meets, joins, and relative and absolute pseudo-complements in $\langle P, \models \rangle$ will allow us to specify an inquisitive semantics for the language of first-order logic which is, from an algebraic perspective, the exact counterpart of classical first-order logic in the inquisitive setting. We will turn to this in Chapter 4. Before that, however, we will consider two additional operations that are particularly natural to perform on propositions in inquisitive semantics.

### 3.2 Projection operators

We noted in Section 2.4.2 that propositions in inquisitive semantics can be seen as inhabiting a two-dimensional space, with non-inquisitive propositions living on one axis and non-informative propositions on the other. Given this picture, it is natural to consider whether it is possible to define general projection operators on this space, i.e., operators that map any given proposition to a corresponding proposition on one of the axes, trivializing either its informative or its inquisitive content. We will refer to such operators as info-cancelling and issue-cancelling projection operators.

Let us first consider more precisely what would be required for an operator $\pi$ to qualify as an issue-cancelling projection operator. Such an operator should project a proposition onto the axis inhabited by non-inquisitive propositions. This means that, when applied to a proposition $P$, $\pi$ should (i) trivialize the inquisitive content of $P$, i.e., turn $P$ into a non-inquisitive proposition, and (ii) preserve the informative content...
of $P$, i.e., yield a proposition that has the same informative content as $P$. This leads us to the following requirements.

**Definition 3.6** (Requirements on issue-cancelling projection operators)
An operator $\pi$ qualifies as an issue-cancelling projection operator just in case for any $P \in \mathcal{P}$:

- $\pi P$ is non-inquisitive
- $\text{info}(\pi P) = \text{info}(P)$

Now, in Section 2.4.2 we saw that if $P$ is non-inquisitive, then we always have that $P = \wp(\text{info}(P))$. This means that in order to satisfy the above requirements, $\pi P$ must amount to $\wp(\text{info}(P))$ for any proposition $P$. Thus, the semantic behavior of $\pi$ is uniquely determined by the given requirements.

**Fact 3.7** (Unique characterization)
An operator $\pi$ qualifies as an issue-cancelling projection operator just in case for any $P \in \mathcal{P}$:

- $\pi P = \wp(\text{info}(P))$

Now let us consider which requirements $\pi$ should fulfill in order to qualify as an info-cancelling projection operator. Such an operator should project a proposition onto the axis inhabited by non-informative propositions. For this, we should require that $\pi$ trivializes the informative content of the proposition to which it applies, i.e., $\pi P$ should always be non-informative. But, given this basic requirement, we cannot further demand that $\pi$ always preserve the inquisitive content of $P$. For, if $P$ and $\pi P$ do not have the same informative content, then their inquisitive content will differ as well.

Fortunately, there is a natural way to overcome this obstacle. Namely, what we can require is that $\pi$ preserve the decision set of $P$, i.e., the set of states that either settle the issue embodied by $P$, or contradict the informative content of $P$ and thereby establish that it is impossible to settle the issue altogether.

**Definition 3.8** (Contradicting and deciding on a proposition)
Let $s$ be an information state and $P$ a proposition. Then we say that:

- $s$ contradicts $P$ just in case $s \cap \text{info}(P) = \emptyset$;
- $s$ decides on $P$ just in case $s$ either supports or contradicts $P$. 
Definition 3.9 (Decision set)
The decision set \( D(P) \) of a proposition \( P \) is the set of states that decide on \( P \).

The decision set of a proposition can be characterized explicitly as follows.

Fact 3.10 (Decision set explicated)
For any proposition \( P \):
- \( D(P) = P \cup P^* \)

Now, what we require of an info-cancelling projection operator \( \pi \) is that, besides trivializing the informative content of the proposition it applies to, it preserves the proposition's decision set. This is a requirement that can in principle be met, since \( P \) and \( \pi P \) can very well have the same decision set even if they differ in informative content.

Definition 3.11 (Requirements on info-cancelling projection operators)
An operator \( \pi \) qualifies as an info-cancelling projection operator just in case for any \( P \in \mathcal{P} \):
- \( \pi P \) is non-informative;
- \( D(\pi P) = D(P) \).

Now suppose that \( \pi \) fulfils these requirements. Then for any \( P \), \( \pi P \) is non-informative, which means that \( \text{info}(\pi P) = W \). But then \( (\pi P)^* = \varnothing(\text{info}(P)) = \varnothing(W) = \varnothing(\emptyset) = \{\emptyset\} \), and therefore \( D(\pi P) = (\pi P) \cup (\pi P)^* = \pi P \). But since \( \pi \) should preserve the decision set of \( P \), we also have that \( D(\pi P) = D(P) = P \cup P^* \). Putting these facts together, we obtain that \( \pi P = P \cup P^* \). Thus, the requirements we placed on \( \pi \) again uniquely determine its behavior.

Fact 3.12 (Unique characterization)
An operator \( \pi \) qualifies as an info-cancelling projection operator just in case for any \( P \in \mathcal{P} \):
- \( \pi P = P \cup P^* \)

Thus, by spelling out the natural requirements on issue-cancelling and info-cancelling projection operators we have arrived at a unique characterization of these operators, which we will denote as \( ! \) and \(?\), respectively.
3.2 Projection Operators

Definition 3.13 (Projection operators)
For any proposition $P$:

- $!P := \varnothing(\text{info}(P))$
- $?P := P \cup P^*$

As depicted in Figure 3.5, the projection operators $!$ and $?$ turn any proposition $P$ into an non-inquisitive proposition $!P$ which has the same informative content as $P$, and a non-informative proposition $?P$ which has the same decision set as $P$. $P$ itself can always be reconstructed as the meet of these two ‘pure components’.

Fact 3.14 (Division)
For any proposition $P$:

- $P = !P \cap ?P$

Finally, let us consider how $?$ and $!$ are related to the algebraic operations identified in Section 3.1. Notice that $?P$ is already explicitly characterized in terms of the algebraic operations: it amounts to the join of $P$ and its absolute pseudo-complement $P^*$. It turns out that $!P$ can also be characterized in terms of pseudo-complementation. Namely, for any proposition $P$, $!P$ amounts to $P^{**}$, i.e., to the proposition that results from two successive applications of the absolute pseudo-complementation operator to $P$.

Fact 3.15 (Projection operators and algebraic operators)
For any proposition $P$:

- $!P = P^{**}$
- $?P = P \cup P^*$
This concludes our discussion of the basic semantic operations that can be performed on propositions in inquisitive semantics. We end this chapter with a brief remark on the linguistic relevance of these operations, which will be further substantiated in later chapters.

### 3.3 Linguistic relevance

Since the algebraic operations on propositions that are associated with the connectives and quantifiers in classical logic are so fundamental, it is to be expected that natural languages will generally have ways to express them as well; just like one would expect, for instance, that basic arithmetic operations like addition and subtraction are generally expressible in natural languages. This makes the algebraic operations discussed here of special interest from a linguistic point of view.

Similar considerations apply to the projection operators. Again, since these semantic operators are so fundamental, it is to be expected that they too are expressible in many natural languages. More specifically, it seems plausible to hypothesize that they are expressed in English and many other languages by declarative and interrogative clause type markers. For instance, on a first approximation, we may hypothesize that declarative clause type marking in English invokes the issue-cancelling projection operator ‘!’; and interrogative clause type marking the info-cancelling projection operator ‘?’; A more detailed account of clause type marking in English in terms of the projection operators will be presented in Chapter 6.

### 3.4 Exercises

**Exercise 3.1 Working through some examples**

Consider the four propositions depicted in Figure 2.4.

1. Determine the absolute pseudo-complement of each of these propositions.

2. Determine the meet and the join of every pair among these propositions.

3. Determine the outcome of applying the projection operators to each of these propositions.
Exercise 3.2  Meets and joins

Prove Facts 3.1 and 3.2. That is, show that every set of propositions in inquisitive semantics has a meet and a join with respect to entailment.

Exercise 3.3  Relative pseudo-complementation

Prove Fact 3.3. That is, show that in inquisitive semantics every proposition \( P \) has a pseudo-complement relative to any other proposition \( Q \), which amounts to \( P \Rightarrow Q = \{ s \mid \text{for every } t \subseteq s, \text{if } t \in P \text{ then } t \in Q \} \).

Exercise 3.4  Projection operators

Suppose we apply both projection operators to a given sentence, one after the other. Does it matter in which order we do this? That is, does the following hold for every proposition \( P \):

\[
?!P = !?P
\]

Exercise 3.5  Projection operators

Show that the projection operators are idempotent, meaning that for every proposition \( P \) we have \( !!P = !P \) and \( ??P = ?P \).

Exercise 3.6  Division

Prove Fact 3.14. I.e., show that for every proposition \( P \) we have \( P = {!P \cap ?P} \).
4

A first-order inquisitive semantics

In this chapter we define an inquisitive semantics for the language of first-order logic, making use of the operations on propositions identified in the previous chapter. We will highlight some of the main features of the system, and illustrate it with a range of examples. In the following chapters we will use this logical framework for the semantic analysis of a number of linguistic constructions.

4.1 Logical language and models

We will consider a standard first-order language $\mathcal{L}$, based on a signature that consists of a set of function symbols $\mathcal{F}_L$ and a set of relation symbols $\mathcal{R}_L$, each with an associated arity $n \geq 0$. As usual, 0-place function symbols will be referred to as individual constants. We assume that the language has $\neg$, $\lor$, $\land$, $\to$, $\exists$, and $\forall$ as its basic logical constants.

We will interpret $\mathcal{L}$ with respect to first-order information models. Such models consist of a set of possible worlds $W$, each associated with a standard first-order model. A standard first-order model, in turn, consists of a domain of individuals $D$ and an interpretation function $I$ which maps any function symbol in $\mathcal{F}_L$ to a function over $D$ and every relation symbol in $\mathcal{R}_L$ to a relation over $D$.

In order to avoid certain issues arising from quantification across different possible worlds, we will restrict our attention to rigid first-order information models, in which the domain of individuals as well as the interpretation of function symbols is fixed across worlds. The only thing that may differ from world to world is the interpretation of relation symbols.

**Definition 4.1** (Rigid first-order information models)
A rigid first-order information model for $\mathcal{L}$ is a triple $(W, D, I)$, where:
• $W$ is a set, whose elements are referred to as *possible worlds*;
• $D$ is a non-empty set, whose elements are referred to as *individuals*;
• $I$ is a map that associates every $w \in W$ with a first-order structure $I_w$ s.t.:
  - for every $w \in W$, the domain of $I_w$ is $D$;
  - for every $n$-ary function symbol $f \in \mathcal{F}_L$, $I_w(f) : D^n \to D$;
    with the condition that for every $w, v \in W$, $I_w(f) = I_v(f)$;
  - for every $n$-ary relation symbol $R \in \mathcal{R}_L$, $I_w(R) \subseteq D^n$.

Unless specified otherwise, we will assume a fixed model throughout our discussion and we will often omit explicit reference to it. So, while in the previous chapters, where we were not yet considering a concrete logical language, we simply assumed a set of possible worlds $W$ as our logical space, we now consider a triple $\langle W, D, I \rangle$, where $W$ is still a set of possible worlds, and the other elements specify the interpretation of the function symbols and relation symbols in our language with regard to these possible worlds.

In order not to have assignments in the way, we will assume that for any $d \in D$, our language $\mathcal{L}$ contains an individual constant $d'$ such that $I_w(d') = d$ for all $w \in W$: if this is not the case, we simply expand the language by adding new constants, and we expand the map $I$ accordingly. In this way we can work only with sentences, i.e., formulas without free variables, and we can do without assignments altogether. This move is of course not essential, but it simplifies notation and terminology considerably.

Finally, it will be convenient to have a notation for the set of worlds in our model in which a given sentence $\varphi$ is classically true, in the standard sense.

**Definition 4.2 (Truth-set)**
For any $\varphi \in \mathcal{L}$, the set of worlds where $\varphi$ is classically true is called the *truth-set* of $\varphi$ and denoted as $|\varphi|$. In particular, for an atomic formula $R(t_1, \ldots, t_n)$:

$$|R(t_1, \ldots, t_n)| = \{ w \in W \mid \langle I_w(t_1), \ldots, I_w(t_n) \rangle \in I_w(R) \}$$

1 The interpretation $I_w(t)$ of a term $t$ is defined inductively as usual: if $t$ is an individual constant $c$, then $I_w(t) = I_w(c)$. If $t = f(t_1, \ldots, t_n)$, then $I_w(t) = I_w(f)(I_w(t_1), \ldots, I_w(t_n))$. Notice that, since we are working only with formulas without free variables, we do not need to consider the case that $t$ is a variable.
4.2 Semantics

We are now ready to recursively associate each sentence $\varphi$ of our first-order language with an inquisitive proposition $[\varphi]$. We will take atomic sentences to behave classically: $R(t_1, \ldots, t_n)$ provides the information that the relation $R$ holds for the individuals $t_1, \ldots, t_n$, and does not raise any issue. Thus, $R(t_1, \ldots, t_n)$ will have as its informative content the set $[R(t_1, \ldots, t_n)]$, and it will not be inquisitive. By Fact 2.19, this implies that $[R(t_1, \ldots, t_n)]$ must amount to $\varphi([R(t_1, \ldots, t_n)])$. As for the connectives and quantifiers, we will take them to express the basic algebraic operations that we identified in Section 3.1.

**Definition 4.3** (First-order inquisitive semantics)

1. $[R(t_1, \ldots, t_n)] := \varphi([R(t_1, \ldots, t_n)])$
2. $[\neg \varphi] := [\varphi]^*$
3. $[\varphi \land \psi] := [\varphi] \cap [\psi]$
4. $[\varphi \lor \psi] := [\varphi] \cup [\psi]$
5. $[\varphi \rightarrow \psi] := [\varphi] \Rightarrow [\psi]$
6. $[\forall x. \varphi(x)] := \bigcap_{d \in D} [\varphi(d')]$
7. $[\exists x. \varphi(x)] := \bigcup_{d \in D} [\varphi(d')]$

We refer to this first-order system as $\text{InqB}$, where B stands for basic. We refer to $[\varphi]$ as the proposition expressed by $\varphi$. The clauses of $\text{InqB}$ constitute a proper inquisitive semantics in the sense that they indeed associate every sentence $\varphi \in \mathcal{L}$ with a proposition in the sense of inquisitive semantics, i.e., a non-empty downward closed set of information states.

**Fact 4.4** (Suitability of the semantics)

For any $\varphi \in \mathcal{L}$, $[\varphi] \in \mathcal{P}$.

All the notions that were introduced in Chapter 2 with reference to propositions can now be formulated with reference to the sentences in our logical language. For instance, we define the informative content of a sentence $\varphi$, $\text{info}(\varphi)$, as the informative content of the proposition it expresses, $\text{info}([\varphi])$. Similarly, the set of alternatives induced by $\varphi$, $\text{alt}(\varphi)$, is the set of alternatives in $\text{alt}([\varphi])$; and the issue raised by $\varphi$ is the issue embodied by $[\varphi]$, which is resolved by an information state $s$ just in case $s \in [\varphi]$. 
Definition 4.5 (Informative content, alternatives, and issues)
For any $\varphi \in \mathcal{L}$:
- $\text{info} (\varphi) := \bigcup \{ \varphi \}$
- $\text{alt} (\varphi) := \text{alt} (\{ \varphi \})$
- The issue raised by $\varphi$ is one that is resolved by a state $s$ just in case $s \in [\varphi]$.

Moreover, we say that one sentence $\varphi$ entails another sentence $\psi$, $\varphi \models \psi$, just in case the proposition expressed by $\varphi$ entails the proposition expressed by $\psi$, and we say that $\varphi$ and $\psi$ are equivalent, $\varphi \equiv \psi$, just in case they express exactly the same proposition.2

Definition 4.6 (Entailment and equivalence)
For any $\varphi, \psi \in \mathcal{L}$:
- $\varphi \models \psi$ just in case $[\varphi] \subseteq [\psi]$
- $\varphi \equiv \psi$ just in case $[\varphi] = [\psi]$

Finally, we say that $\varphi$ is true in a world $w$ in case the proposition it expresses is true in $w$, i.e., $w \in \text{info} (\varphi)$; and we say that $\varphi$ is supported by an information state $s$, $s \models \varphi$, in case the proposition it expresses is supported by $s$, i.e., $s \in [\varphi]$.

Definition 4.7 (Truth and support)
For any $\varphi \in \mathcal{L}$:
- $\varphi$ is true in $w$ if and only if $w \in \text{info} (\varphi)$
- $\varphi$ is supported by $s$, notation $s \models \varphi$, if and only if $s \in [\varphi]$

Notice that, just like the proposition expressed by $\varphi$ in classical logic is the set of worlds where $\varphi$ is true, the proposition expressed by $\varphi$ in $\text{InqB}$ is the set of states where $\varphi$ is supported. As a consequence of this fact, $\text{InqB}$ is completely characterized by the support conditions of the sentences in the language. These support conditions are as follows.

Fact 4.8 (Support conditions)
1. $s \models R(t_1, \ldots, t_n)$ if and only if $s \subseteq |R(t_1, \ldots, t_n)|$
2. $s \models \neg \varphi$ if and only if $\forall t \subseteq s : t \neq \emptyset \text{ then } t \not\models \varphi$
3. $s \models \varphi \wedge \psi$ if and only if $s \models \varphi$ and $s \models \psi$

---

2 Notice that the notions of entailment and equivalence given here are semantic notions which assume a given information model; as such, they incorporate facts that are encoded by the model as analytical, i.e., true in all possible worlds, but which are not purely logical. The purely logical notion of entailment—studied in inquisitive logic—is obtained by universally quantifying over all information models.
In much work on inquisitive semantics (Ciardelli, 2009; Groenendijk and Roelofsen, 2009; Ciardelli and Roelofsen, 2011; Ciardelli et al., 2015; Ciardelli, 2016d, 2018), InqB is in fact characterized directly in terms of support conditions. The proposition expressed by a sentence is then defined in terms of these support conditions, i.e., as the set of all states that support the sentence. An advantage of this approach is that it parallels the usual presentation of classical logic, with truth conditions as the basic notion. Another advantage, at least for certain purposes, is that it allows for a very efficient presentation of the system, bypassing many of the more abstract notions that we introduced here before even starting to consider a concrete logical language. In this book, the support-based perspective will play a key role in Chapter 8, allowing for a perspicuous presentation of the inquisitive account of propositional attitudes.

There are two main reasons why we have chosen a less direct route here, following Ciardelli et al. (2013a) and Roelofsen (2013a). First, the current presentation of the new inquisitive notion of propositions (Chapter 2) brings out very explicitly how the standard information-centred notion of semantic content is enriched, why the new notion is shaped exactly the way it is, and that it naturally allows for various further extensions and refinements (see the references in section 2.6 as well as in the Further Reading section.) Second, the algebraic perspective adopted here (Chapter 3) makes it possible to motivate the treatment of the connectives and quantifiers in InqB in a solid way, relying only on the structure of our new space of propositions. Moreover, it shows that InqB is, in a very precise sense, the exact counterpart of classical logic in the inquisitive setting. Thus, unlike a support-based exposition, this mode of presentation flows directly from the abstract motivations and conceptual underpinnings of the system to its concrete implementation.

### 4.3 Semantic categories and projection operators

We say that a sentence is informative, inquisitive, a hybrid, or a tautology just in case the proposition that it expresses is. This amounts to the following.

4. \( s \models \phi \lor \psi \iff s \models \phi \text{ or } s \models \psi \)

5. \( s \models \phi \rightarrow \psi \iff \forall t \subseteq s : \text{if } t \models \phi \text{ then } t \models \psi \)

6. \( s \models \forall x \phi(x) \iff s \models \phi(d') \text{ for all } d \in D \)

7. \( s \models \exists x \phi(x) \iff s \models \phi(d') \text{ for some } d \in D \)
Definition 4.9 (Semantic categories)
We say that a sentence $\varphi \in L$ is:

- informative iff $\text{info}(\varphi) \neq W$.
- inquisitive iff $\text{info}(\varphi) \not\subseteq [\varphi]$.
- a hybrid iff it is both informative and inquisitive;
- a tautology iff it is neither informative nor inquisitive.

Fact 4.10 (Direct characterization of trivial sentences)

- $\varphi$ is non-inquisitive $\iff [\varphi] = \varphi(\text{info}(\varphi))$ $\iff [\varphi]$ has a greatest element.
- $\varphi$ is non-informative $\iff \text{info}(\varphi) = W$.
- $\varphi$ is a tautology $\iff [\varphi] = \varphi(W)$.

In Section 3.2 we characterized two projection operators on propositions, which trivialize either the informative or the inquisitive content of any given proposition. Now that we are considering a concrete logical language, we will introduce two one-place connectives that express these projection operators. We will denote these connectives as $!$ and $?$, just like the operators they express.

Definition 4.11 (Projection operators)
For any $\varphi \in L$:

- $[!\varphi] := ![\varphi]
- [?\varphi] := [?\varphi]

Recall from Fact 3.15 that the projection operators on propositions can be characterized algebraically:

- $!P = P^{**}
- ?P = P \cup P^{*}$

Since negation expresses absolute pseudo-complementation and disjunction expresses the join operation, this means that the connectives $!$ and $?$ can be characterized in terms of negation and disjunction.

Fact 4.12 (Projection operators in terms of negation and disjunction)
For any $\varphi \in L$:

- $!\varphi \equiv \neg \neg \varphi$
- $?\varphi \equiv \varphi \lor \neg \varphi$
This means that ! and ? do not have to be added to our logical language as primitive connectives; !φ and ?φ can simply be regarded as abbreviations of ¬¬φ and φ ∨ ¬φ, respectively.

Finally, we have that a sentence φ is always equivalent to the conjunction of its two 'pure components' !φ and ?φ (the analogue of Fact 3.14).

Fact 4.13 (Division)
For any φ:

• φ ≡ !φ ∧ ?φ

4.4 Examples

Now let us consider some concrete sentences in \( \text{InqB} \) and the propositions that they express. We will assume that our language contains just one unary predicate symbol, \( R \), and two individual constants, \( a \) and \( b \). Accordingly, we will assume that the domain of discourse consists of just two objects, denoted by \( a \) and \( b \), respectively. Our logical space consists of four worlds, one in which both \( Ra \) and \( Rb \) are true, one in which \( Ra \) is true but \( Rb \) is false, one in which \( Rb \) is true but \( Ra \) is false, and one in which neither \( Ra \) nor \( Rb \) is true. These worlds will be labeled 11, 10, 01, and 00, respectively. As usual, in order to keep the pictures orderly we display only the maximal elements of a proposition. For concreteness, we will read \( Ra \) as 'Ann is in Rome', and \( Rb \) as 'Bob is in Rome'.

Atomic sentences. Let us first consider the proposition expressed by the atomic sentences \( Ra \) and \( Rb \). According to the clause for atomic sentences, \([Ra]\) consists of all states \( s \) such that every world in \( s \) makes \( Ra \) true, i.e., the state \{11, 10\} and all substates thereof. Thus, as depicted in Figure 4.1(a), \([Ra]\) has a greatest element, \{11, 10\}. Fact 4.10 therefore ensures that \( Ra \) is non-inquisitive. It provides the information that Ann is in Rome, and it does not request any further information. So it behaves just as in classical logic. Analogously, \( Rb \) provides the information that Bob is in Rome, without requesting any further information. The proposition expressed by \( Rb \) is depicted in Figure 4.1(b).

Disjunction. Next, consider the disjunction \( Ra \lor Rb \). According to the clause for disjunction, \([Ra \lor Rb]\) consists of those states that are in \([Ra]\) or in \([Rb]\). These are \{11,10\}, \{11,01\}, and all substates thereof, as depicted in Figure 4.1(c).
Since $\text{info}(Ra \lor Rb) = \bigcup [Ra \lor Rb] = \{11, 10, 01\} \neq W$, the disjunction $Ra \lor Rb$ is informative. It provides the information that at least one of Ann and Bob is in Rome. However, unlike in the case of atomic sentences, in this case there is no unique greatest element in $[Ra \lor Rb]$ that includes all the others. Instead, there are two maximal elements, $[Ra] = \{11, 10\}$ and $[Rb] = \{11, 01\}$, which together contain all the others. Thus, besides being informative, $Ra \lor Rb$ is also inquisitive.

In order to settle the issue that it raises, one has to establish either that Ann is in Rome, or that Bob is in Rome.

A note of caution is perhaps in order here: it is important to keep in mind that $\text{InqB}$ does not directly embody an analysis of sentences in natural language: it only provides the tools to formulate such analyses. In particular, a disjunctive sentence in $\text{InqB}$ like $Ra \lor Rb$ does not necessarily correspond to a disjunctive declarative sentence in English like (1) below, or to a disjunctive interrogative sentence like (2) for that matter.

(1) Ann is in Rome or Bob is in Rome.

(2) Is Ann in Rome, or is Bob in Rome?

In Chapter 6 we will present a concrete analysis of such sentences using $\text{InqB}$. On that analysis, (1) corresponds to $!(Ra \lor Rb)$ and (2) corresponds either to $?((Ra \lor Rb)$ or to $Ra \lor Rb$, depending on intonation.

**Negation.** Let us now turn to negation. According to the clause for negation, $[\neg Ra]$ consists of all states $s$ such that $s$ does not have any world in common with any state in $[Ra]$. Thus, $[\neg Ra]$ consists of all states that do not contain the worlds 11 and 10, which are $|\neg Ra| = \{01, 00\}$ and all substates thereof, as depicted in Figure 4.1(d). Since this set of states has a greatest element, Fact 4.10 ensures that $\neg Ra$ is non-inquisitive. It provides the information that Ann is not in Rome, and does not request any further information.

Now let us consider the negation of an inquisitive disjunction, $\neg (Ra \lor Rb)$. According to the clause for negation, $[\neg (Ra \lor Rb)]$ consists
of all states which do not have a world in common with any state in \([Ra \lor Rb]\). Thus, \([\neg(Ra \lor Rb)]\) consists of all states that do not contain the worlds 11, 10, and 01, which are \{00\} and \emptyset, as depicted in Figure 4.1(e). Again, there is a unique maximal element, namely \(|\neg(Ra \lor Rb)| = \{00\}. Thus, \neg(Ra \lor Rb) provides the information that neither Ann nor Bob is in Rome, just like in classical logic, and does not request any further information.

These examples of negative sentences exemplify the general observation that we made above concerning pseudo-complementation (just below Fact 3.4): the absolute pseudo-complement of a proposition always contains a greatest element. This means that a negative sentence \neg \phi is never inquisitive; it simply provides the information that \phi is false, and does not request any further information.

**Projection operators.** Next let us consider \(! (Ra \lor Rb)\), which abbreviates \!\neg(Ra \lor Rb). We have just seen that \neg(Ra \lor Rb) expresses the proposition depicted in Figure 4.1(e). Applying negation again, we arrive at the proposition depicted in Figure 4.2(a), which has \[Ra \lor Rb\] as its unique alternative. Notice that \!(Ra \lor Rb) is not equivalent with \(Ra \lor Rb\). The two sentences have the same informative content, but the former is purely informative, while the latter is also inquisitive. This exemplifies the general nature of \!: for any sentence \phi, \!\phi is a non-inquisitive proposition with the same informative content as \phi. If \phi itself is already non-inquisitive, then \!\phi and \phi are equivalent; if \phi is inquisitive, as in the example just considered, the two express different propositions.

Let us now turn to ?. Consider \(?Ra\), which is an abbreviation of \(Ra \lor \neg Ra\). We have already seen what \[Ra\] and \[\neg Ra\] are. According to the clause for disjunction, \[?Ra\] = \[Ra \lor \neg Ra\] consists of all states that are either in \[Ra\] or in \[\neg Ra\]. These states are \[Ra\], \[\neg Ra\], and all substates thereof, as depicted in Figure 4.2(b). Since \text{info}(?Ra) = W, \?Ra is non-informative. On the other hand, since \[?Ra\] contains two alternatives, it is inquisitive. In order to settle the issue that it raises, one has to establish either that Ann is in Rome, or that Ann is not in Rome.

![Figure 4.2 Projection operators.](image-url)
That is, one has to establish whether Ann is in Rome. Thus, while ?Ra is shorthand for Ra ∨ ¬Ra, perhaps the most famous classical tautology, it is not a tautology in InqB: instead, it corresponds to the polar question ‘whether Ra’. Analogously, ?Rb, depicted in Figure 4.2(c), corresponds to the polar question ‘whether Rb’.

If ? applies to the disjunction Ra ∨ Rb, which is already inquisitive, then it yields the proposition depicted in Figure 4.2(d). [Ra ∨ Rb] already contains two alternatives, [Ra] and [Rb]; ? adds a third alternative, which is the set of worlds that are neither in [Ra] nor in [Rb]. Thus, in order to resolve the issue raised by ?(Ra ∨ Rb), one has to establish either that Ann is in Rome, or that Bob is, or that neither is.

Finally, let us consider a case where ! and ? both apply, one after the other: ?!(Ra ∨ Rb). As we already saw above, ![Ra ∨ Rb] contains a single alternative, consisting of all worlds where at least one of a and b is in Rome. As depicted in Figure 4.2(e), ? adds a second alternative, which is the set of worlds where neither Ann nor Bob is in Rome. Notice that the resulting proposition differs from that expressed by ?(Ra ∨ Rb), which contains three alternatives rather than two. In order to settle the issue expressed by ?!(Ra ∨ Rb) it is sufficient to establish that at least one of Ann and Bob is in Rome. In order to settle the issue expressed by ?(Ra ∨ Rb) this is not sufficient; rather, it needs to be established for one of Ann and Bob that he or she is in Rome, or that neither of them is. We will see in Chapter 6 that the ability to capture such subtle differences is crucial in order to account for various kinds of disjunctive questions in natural languages.

Conjunction. Next, let us consider conjunction. First, let us look at the conjunction of our two atomic, non-inquisitive sentences, Ra and Rb. According to the clause for conjunction, [Ra ∧ Rb] consists of those states that are both in [Ra] and in [Rb]. These are {11} and ∅. Thus, [Ra ∧ Rb] has a greatest element, namely {11}, and accordingly Ra ∧ Rb provides the information that both Ann and Bob are in Rome, just like in the classical case, and does not request any further information.

![Figure 4.3](image-url)
Now let us look at the conjunction of two inquisitive sentences, $Ra$ and $Rb$. As depicted in Figure 4.3(b), the proposition $[Ra \land Rb]$ contains four alternatives, $|Ra \land Rb|$, $|Ra \land \neg Rb|$, $|\neg Ra \land Rb|$, and $|\neg Ra \land \neg Rb|$. Since these alternatives together cover the entire logical space $Ra \land Rb$ is non-informative. On the other hand, since there is more than one alternative, $Ra \land Rb$ is inquisitive. In order to settle the issue that it raises, one has to establish one of $Ra \land Rb$, $Ra \land \neg Rb$, $\neg Ra \land Rb$, $\neg Ra \land \neg Rb$. Thus, our conjunction is a purely inquisitive sentence which requests enough information to settle both the issue whether Ann is in Rome, contributed by $Ra$, and the issue whether Bob is in Rome, contributed by $Rb$.

These two examples of conjunctive sentences exemplify a general fact: if $\phi$ and $\psi$ are non-inquisitive, then the conjunction $\phi \land \psi$ is non-inquisitive as well, conveying both the information provided by $\phi$ and the information provided by $\psi$; on the other hand, if $\phi$ and $\psi$ are non-informative, then the conjunction $\phi \land \psi$ is non-informative as well, expressing an issue which is settled just in case both the issue expressed by $\phi$ and the one expressed by $\psi$ are settled.

**Implication.** Next, let us consider implication. Again, we will first consider a simple case, $Ra \rightarrow Rb$, where both the antecedent and the consequent are atomic, and therefore non-inquisitive. According to the clause for implication, $[Ra \rightarrow Rb]$ consists of all states $s$ such that every substate $t \subseteq s$ that is in $[Ra]$ is also in $[Rb]$. These are all and only those states that are contained in $|Ra \rightarrow Rb| = \{11,01,00\}$, as depicted in Figure 4.3(c). So, $[Ra \rightarrow Rb]$ has a unique greatest element, $|Ra \rightarrow Rb|$, which means that the implication $Ra \rightarrow Rb$ is a non-inquisitive sentence which, just like in the classical setting, provides the information that if Ann is in Rome, then so is Bob.

Now let us consider a more complex case, $Ra \rightarrow ?Rb$, where the consequent is non-informative but inquisitive. As depicted in Figure 4.3(d), the proposition $[Ra \rightarrow ?Rb]$ contains two alternatives, $|Ra \rightarrow Rb| = \{11,01,00\}$, and $|Ra \rightarrow \neg Rb| = \{10,01,00\}$. Since these two alternatives together cover the entire logical space, our implication is non-informative. Moreover, since there is more than one alternative, the implication is inquisitive. In order to settle the issue that it raises, one must either establish $Ra \rightarrow Rb$, or $Ra \rightarrow \neg Rb$. In the former case one establishes that if Ann is in Rome, then so is Bob; in the latter case, that if Ann is in Rome, then Bob isn't. So $Ra \rightarrow ?Rb$ requests enough information to establish whether Bob is in Rome under the assumption that Ann is.
Again, these two examples of conditional sentences exemplify a general feature of \(\text{InqB}\): if \(\psi\) is non-inquisitive, then \(\varphi \rightarrow \psi\) is non-inquisitive as well; and similarly, if \(\psi\) non-informative, then \(\varphi \rightarrow \psi\) is non-informative as well.

Quantification. Finally, let us consider existential and universal quantification. As usual, existential quantification behaves essentially like disjunction and universal quantification behaves essentially like conjunction. In fact, since our current domain of discourse consists of only two objects, denoted by \(a\) and \(b\), respectively, \(\exists x. Rx\) expresses exactly the same proposition as \(Ra \lor Rb\), depicted in Figure 4.1(c), and \(\forall x. Rx\) expresses exactly the same proposition as \(Ra \land Rb\), depicted in Figure 4.3(a). Finally, consider the proposition expressed by \(\forall x. ?Rx\), depicted in Figure 4.3(e). Notice that this proposition induces a partition on the logical space, where each block of the partition consists of worlds that agree on the extension of \(R\). Thus, \(\forall x. ?Rx\) asks for an exhaustive specification of the individuals that are in Rome.

4.5 Informative content, truth, and support

Recall that \(\text{info}(\varphi)\) is defined as \(\bigcup [\varphi]\), which is a set of worlds. In classical logic, the informative content of a sentence \(\varphi\) is also embodied by a set of worlds, namely the set of all worlds where \(\varphi\) is true, \(|\varphi|\). Thus, it is natural to ask how these two notions of informative content relate to each other. The answer is that the relation is as direct as it could be: the two always coincide.

**Fact 4.14** (Informative content and truth)
For any \(\varphi \in \mathcal{L}\), \(\text{info}(\varphi) = |\varphi|\).

This shows that \(\text{InqB}\) preserves the classical treatment of informative content. The system only differs from classical logic in that, besides informative content, it takes inquisitive content into consideration as well.

Notice that Facts 2.18 and 4.14 together yield the following characterization of non-informative and non-inquisitive sentences in terms of classical truth.

**Fact 4.15** (Informational and inquisitive triviality in terms of classical truth)

- \(\varphi\) is non-informative \(\iff |\varphi| = W\)
- \(\varphi\) is non-inquisitive \(\iff |\varphi| \in [\varphi] \iff [\varphi] = \varphi(|\varphi|)\)
Thus, non-informative sentences in $\text{InqB}$ are precisely those sentences that are classically true at any world. On the other hand, a sentence $\varphi$ is non-inquisitive in $\text{InqB}$ just in case the proposition it expresses is fully determined by its classical truth-set: it provides the information that $\varphi$ is true, and does not request any further information. Thus, non-inquisitive sentences behave exactly as in classical logic.

The classical behavior of non-inquisitive sentences results in a tight connection between their support conditions and their truth conditions. Namely, such a sentence $\varphi$ is supported by a state $s$ just in case it is true in every world in $s$. This holds only for non-inquisitive sentences; the moment a sentence becomes inquisitive, the connection between support and truth breaks down.

**Fact 4.16 (Support and truth)**
The following are equivalent for any sentence $\varphi \in \mathcal{L}$:

- $\varphi$ is non-inquisitive
- For every information state $s$:
  
  $s \models \varphi \iff \varphi$ is true in every world in $s$

### 4.6 Syntactic properties of non-hybrid sentences

Below we provide some syntactic conditions which make it easy to recognize sentences that are either non-informative or non-inquisitive, just based on their form, without inspecting their meaning.

Let us start with non-inquisitive sentences. The following fact provides some syntactic conditions which guarantee that a sentence is non-inquisitive. These conditions generalize some of the more specific observations that were already made in discussing the examples above.

**Fact 4.17 (Sufficient conditions for non-inquisitivy)**

1. Atomic sentences are always non-inquisitive;
2. $\neg \varphi$ is always non-inquisitive;
3. $!\varphi$ is always non-inquisitive;
4. If $\varphi$ and $\psi$ are non-inquisitive, then so is $\varphi \land \psi$;
5. If $\psi$ is non-inquisitive, then so is $\varphi \rightarrow \psi$ for any antecedent $\varphi$;
6. If $\varphi(d')$ is non-inquisitive for all $d \in D$, then so is $\forall x \varphi(x)$.

Now let us turn to non-informative sentences. Again we provide some syntactic conditions that guarantee that a given sentence is non-informative, generalizing some of the more specific observations made in discussing the examples in Section 4.4.
Fact 4.18 (Sufficient conditions for non-informativity)

1. \( ?\phi \) is always non-informative;
2. If \( \phi \) and \( \psi \) are non-informative, so is \( \phi \land \psi \);
3. If \( \psi \) is non-informative, then so are \( \phi \lor \psi \) and \( \phi \rightarrow \psi \), for any \( \phi \);
4. If \( \phi(d') \) is non-informative for all \( d \in D \), then so is \( \forall x\phi(x) \);
5. If \( \phi(d') \) is non-informative for some \( d \in D \), then so is \( \exists x\phi(x) \).

4.7 Sources of inquisitiveness

The partial syntactic characterization of non-inquisitive sentences in Fact 4.17 implies that disjunction, the existential quantifier, and the projection operator \( ? \) are the only sources of inquisitiveness in our logical language.

Fact 4.19 (Sources of inquisitiveness)

Any sentence that does not contain \( \lor \), \( \exists \), or \( ? \) is non-inquisitive.

Note that there is a close connection between disjunction, the existential quantifier, and the \( ? \) operator in \text{InqB}. Namely, they all behave as join operators: \( \phi \lor \psi \) is the join of \( [\phi] \) and \( [\psi] \), \( \exists x\phi(x) \) is the join of \( \{[\phi(d')] \mid d \in D\} \), and \( ?\phi \) is the join of \( [\phi] \) and \( [\phi]^* \). In terms of semantic operators, then, the join operator is the essential source of inquisitiveness: without applying this operator, it is impossible to produce inquisitive propositions from non-inquisitive ones.

This fact may provide the basis for an explanation of the well-known observation that in many natural languages, question words are homophonous with words for disjunction and/or existentials (see Jayaseelan, 2001; Bhat, 2005; Haida, 2007; Jayaseelan, 2008; Cable, 2010; AnderBois, 2011; Slade, 2011, among others). For instance, Malayalam -oo and Japanese \( ka \) are used for all three purposes.

<table>
<thead>
<tr>
<th>Malayalam</th>
<th>Japanese</th>
<th>English</th>
</tr>
</thead>
<tbody>
<tr>
<td>Existential</td>
<td>aar-oo</td>
<td>dare-ka</td>
</tr>
<tr>
<td>Disjunction</td>
<td>Anna-oo Peter-oo</td>
<td>Anna-ka Peter-ka</td>
</tr>
<tr>
<td>Question</td>
<td>Anna wanna-(w)oo</td>
<td>Anna wa kita-ka</td>
</tr>
</tbody>
</table>

Szabolcsi (2015b) proposes an account of this cross-linguistic phenomenon in inquisitive semantics, suggesting that the inquisitive join operation can indeed be seen as the semantic common core of disjunctive, existential, and interrogative constructions in languages like Malayalam and Japanese.
4.8 Comparison with alternative semantics

There is a close connection between the treatment of disjunction and existentials in InqB, and their treatment in alternative semantics (Kratzer and Shimoyama, 2002; Menéndez-Benito, 2005; Simons, 2005; Alonso-Ovalle, 2006; Aloni, 2007, among others). In both frameworks, disjunction and existentials introduce sets of alternatives. In the case of alternative semantics, this treatment is motivated by a number of empirical phenomena, including free choice inferences, exclusivity implicatures, and counterfactual conditionals with disjunctive antecedents. The analysis of disjunction and existentials as introducing sets of alternatives has made it possible to develop new accounts of these phenomena which improve considerably on previous accounts. However, while work on alternative semantics has provided ample empirical motivation for its treatment of disjunction and existentials, its explanatory power would increase substantially if the treatment could be motivated by considerations independent of the linguistic phenomena that it has aimed to capture.

Moreover, the empirical phenomena that have motivated work on disjunction and existentials in alternative semantics have been taken to require a radical departure from the classical algebraic treatment of disjunction and existentials. For instance, Alonso-Ovalle (2006) writes in the conclusion section of his dissertation:

This dissertation has investigated the interpretation of counterfactuals with disjunctive antecedents, unembedded disjunctions, and disjunctions under the scope of modals. We have seen that capturing the natural interpretation of these constructions proves to be challenging if the standard analysis of disjunction, under which or is the Boolean join, is assumed.

Similarly, Simons (2005) starts her paper as follows:

In this paper, the meanings of sentences containing the word or and a modal verb are used to arrive at a novel account of the meaning of or coordinations. It has long been known that such sentences […] pose a problem for the standard treatment of or as a Boolean connective equivalent to set union.

The approach we have taken here shows that, once we take both informative and inquisitive content into account, general algebraic considerations lead essentially to the treatment of disjunction that was proposed in alternative semantics, thus providing exactly the independent motivation that has so far been missing (for detailed discussion of this point, see Roelofsen, 2015b). Moreover, it shows that the treatment
of disjunction as generating sets of alternatives can actually be seen as a natural generalization of the classical treatment, rather than a radical departure from it: as soon as we adopt a notion of meaning that encompasses both informative and inquisitive content, treating disjunction as a join operator automatically gives it the potential to generate multiple alternatives. Thus, we can have our cake and eat it: we can treat disjunction as a join operator and as introducing sets of alternatives at the same time. In inquisitive semantics, the two go hand in hand.3

### 4.9 Exercises

**Exercise 4.1 Propositions in InqB**

Using diagrams analogous to those in Figure 4.1, depict the propositions expressed by the following formulas:

1. \( Ra \land ?Rb \)
2. \( ?Ra \lor ?Rb \)
3. \( \neg(Ra \land Rb) \)
4. \( !\exists xRx \rightarrow \exists xRx \)
5. \( !\exists xRx \rightarrow \forall x?Rx \)
6. \( ?Ra \rightarrow ?Rb \)

**Exercise 4.2 De Morgan’s laws**

Below are two well-known classical equivalences, known as De Morgan’s laws:

\[
\neg(\varphi \lor \psi) \equiv \neg\varphi \land \neg\psi \\
\neg(\varphi \land \psi) \equiv \neg\varphi \lor \neg\psi
\]

Do these equivalences also hold in inquisitive semantics? If yes, give a proof. If no, provide a counterexample.

---

3 It should be noted that, while both in alternative semantics and in inquisitive semantics disjunction generates alternatives in a similar way, there is also a subtle but important difference. Namely, in inquisitive semantics one alternative can never be nested in another, unlike in alternative semantics. This has certain advantages, as we will discuss briefly in Section 9.1 (for more detailed discussion of this difference, see Ciardelli and Roelofsen, 2017a).
Exercise 4.3 The law of double negation

Recall that in classical logic, $\neg\neg \varphi \rightarrow \varphi$ is a tautology for any given formula $\varphi$. Show that, in InqB, $\neg\neg \varphi \rightarrow \varphi$ is a tautology if and only if $\varphi$ is non-inquisitive.

Explain why this difference between classical logic and InqB arises, even though $\neg$ and $\rightarrow$ express exactly the same algebraic operations in both frameworks (absolute and relative pseudo-complementation, respectively).
5

Questions

In Chapters 2–4 we laid out the basic architecture of inquisitive semantics. In the present chapter and the ones to follow we will discuss its applications to the analysis of natural language, and its relations to other semantic frameworks. In this chapter we discuss how and to what extent the semantics of various types of questions in English can be captured in $\text{InqB}$. We start in Sections 5.1–5.5 by examining a number of classes of natural language questions, and discussing the corresponding $\text{InqB}$-translations. In Section 5.6 we point out some features of the meaning of questions which are not captured by these translations, and we briefly discuss how $\text{InqB}$ could be extended to capture these features, providing pointers to the literature in which such extensions are pursued.

Other important elements of the analysis of questions in inquisitive semantics will be put in place in the following chapters. In Chapters 6 and 7 we will see how the $\text{InqB}$-translation of certain classes of questions and statements can be built up compositionally. In Chapter 8 we will illustrate how the analysis of questions given here can be combined with suitable inquisitive entries for attitude verbs like know and wonder to obtain an analysis of question embedding. Finally, in Chapter 9 we will compare our approach to other influential approaches to question semantics.

5.1 Polar questions

Polar questions are questions that ask for the truth-value of a given proposition, as exemplified in (1).

(1) Is Alice married to Bob? $?\text{Mab}$

The issue expressed by (1) is resolved in an information state $s$ if it follows from the information available in $s$ that Alice is married to Bob (i.e., if $s \subseteq |\text{Mab}|$) or if it follows that Alice is not married to Bob (i.e.,
Questions

if $s \subseteq \lbrack Mab \rbrack$. This means that the issue expressed by (1) is precisely the issue expressed by the $\text{InqB}$-sentence $\lbrack ?Mab \rbrack$, which can thus be regarded as an $\text{InqB}$-translation of the English question in (1).\footnote{We will use the following notation throughout this chapter: for any set of information states $P$, we will write $P^\downarrow$ for the set of information states that are contained in some element of $P$, i.e., $P^\downarrow := \{ s \mid s \subseteq t \text{ for some } t \in P \}$.}

(2) \[ \lbrack (1) \rbrack = \lbrack ?Mab \rbrack \]
\[ = \lbrack \lbrack Mab \rbrack, \lbrack Mab \rbrack \rbrack^\downarrow \]

5.2 Alternative questions

Alternative questions are questions that list a number of options, separated by disjunction, and ask for a choice among these. An example is given in (3), where the arrows $\uparrow$ and $\downarrow$ each mark the end of an intonational phrase and indicate whether the intonation is rising of falling at the end of that phrase (different intonation patterns for disjunctive questions will be briefly discussed below and in more detail in the next chapter).

(3) Is Alice married to Bob$^\uparrow$ or to Charlie$^\downarrow$? \hspace{1cm} $Mab \lor Mac$

The issue expressed by (3) is resolved in an information state $s$ if it follows from the information available in $s$ that Alice is married to Bob (i.e., if $s \subseteq \lbrack Mab \rbrack$) or if it follows that she is married to Charlie (i.e., $s \subseteq \lbrack Mac \rbrack$). This means that the issue expressed by (3) is the one that is expressed by the $\text{InqB}$-sentence $Mab \lor Mac$.

(4) \[ \lbrack (3) \rbrack = \lbrack Mab \lor Mac \rbrack \]
\[ = \lbrack \lbrack Mab \rbrack, \lbrack Mac \rbrack \rbrack^\downarrow \]

Notice in particular that the disjunction word or occurring in (3) is translated as the connective $\lor$ in $\text{InqB}$.

One concern with translating (3) in this way is that, in our terminology, the formula $Mab \lor Mac$ is not only inquisitive, but also informative. Its informative content is that Alice is married to either Bob or Charlie. However, in uttering the question in (3), a speaker does not present the information that Alice is married to either Bob or Charlie as new information, but rather as something that she presupposes (see, e.g., Karttunen and Peters, 1976; Biezma and Rawlins, 2012). Properly capturing this requires an extension of the basic $\text{InqB}$
framework which, besides informative and inquisitive content, allows us to represent presuppositional content as well. We refer to AnderBois (2012), Ciardelli et al. (2012), and Roelofsen (2015a) for such an extension of the framework, and a refined representation of the meaning of alternative questions like (3).

A special case of an alternative question is obtained by disjoining a clause with its own negation, as in (5).

(5) Is Alice married to Bob↑ or not↓? \( M_a b \lor \neg M_a b \)

In this case, the translation is \( M_a b \lor \neg M_a b \), which is equivalent with the translation that we provided for the polar question in (1), \( \? M_a b \). This is expected, since the two questions indeed express the same issue.

Now consider the question in (6), which only differs from (3) in intonation: there is no intonational phrase boundary after the first disjunct, i.e., the two disjuncts are pronounced in a single intonational phrase, and the final pitch contour is rising rather than falling.

(6) Is Alice married to Bob or to Charlie↑? \( \? ! (M_a b \lor M_c) \)

With this intonation pattern, the question is interpreted as a polar question, asking whether it is true or false that Alice is married to Bob or Charlie. That is, the issue expressed by (6) is resolved in an information state \( s \) if it follows from the information available in \( s \) that Alice is married to Bob or Charlie (i.e., if \( s \subseteq |M_a b \lor M_c| \)) or if it follows that she is not married to either Bob or Charlie (i.e., \( s \subseteq \overline{|M_a b \lor M_c|} \)). This means that the issue expressed by (6) is precisely the one that is expressed by the InqB-sentence \( \? ! (M_a b \lor M_c) \).

(7) \([6] = \? ! (M_a b \lor M_c) \]
\( = \{|M_a b \lor M_c|, \overline{|M_a b \lor M_c|}\} \uparrow \)

## 5.3 Open disjunctive questions

There exists yet another intonational variant of (3), given in (8) below.

(8) Context: Susan, who is in her sixties, decided to organize a high school reunion. She is now sending out invitation letters. In school, Susan was close friends with Alice, Bob, Charlie, and Drew. After school, she moved to Spain and only kept in touch with Drew. She heard long ago that Alice ended up marrying someone from their class, but she doesn’t know who exactly. In case Alice got
married to Bob or Charlie, Susan wants to write an invitation letter to Alice and Bob/Charlie together, and say something nice about their marriage. In case Alice got married to someone else, she will just send her an individual invitation letter and won’t mention her marriage. So she calls Drew and asks her:

Is Alice married to Bob† or to Charlie†? 

Questions of this kind are called open disjunctive questions (Roelofsen and van Gool, 2010; Roelofsen and Farkas, 2015). Note that the two disjuncts in (8) are pronounced in two separate intonational phrases, as in the alternative question in (3) and unlike in the polar question in (6). However, the final pitch contour is rising, as in the polar question in (6) and unlike in the alternative question in (3). Thus, the intonation pattern of open disjunctive questions differs both from that of alternative questions and from that of polar disjunctive questions.

The same holds for their resolution conditions. The issue expressed by (8) is resolved in an information state $s$ if (i) it follows from the information available in $s$ that Alice is married to Bob (i.e., if $s \subseteq [\text{Mab}]$), or (ii) it follows that she is married to Charlie (i.e., $s \subseteq [\text{Mac}]$), or (iii) it follows that she is not married to either Bob or Charlie (i.e., $s \subseteq [\text{Mab} \lor \text{Mac}]$). This means that the issue expressed by (8) is the one expressed by the $\text{InqB}$-sentence $?(\text{Mab} \lor \text{Mac})$.

\[(8) \quad [(8)] = [?(\text{Mab} \lor \text{Mac})] \]
\[= \{ [\text{Mab}], [\text{Mac}], [\text{Mab} \lor \text{Mac}] \}^1 \]

Note in particular that an information state in which it is established that Alice is married to either Bob or Charlie, but not to which of the two, does not contain enough information to settle the issue expressed by (8), while it does settle the issue expressed by the polar question in (6). Also note that (8) does not presuppose that Alice is married to either Bob or Charlie, unlike the alternative question in (3).²

5.4 Wh-questions

Besides polar questions, alternative questions, and open disjunctive questions, another major class of questions occurring in natural

² We refer to Roelofsen and Farkas (2015) for further examples of open disjunctive questions, and further discussion of how they differ from alternative and polar disjunctive questions. In particular, besides in intonation and in resolution conditions, the three question-types also differ in the extent to which they license yes/no responses.
languages consists of wh-questions. Below we will discuss several prominent kinds of wh-questions, exemplified in (10).

(10) a. Who did Alice invite to her birthday party?
   b. What is something that Alice really likes?
   c. Who is Alice married to?
   d. Who is married to whom?
   e. Which students did Alice invite to her party?

First, we will distinguish mention-all questions such as (10a) from mention-some questions such as (10b) and single-match questions such as (10c). Then we will turn to questions with multiple wh-words, such as (10d), and ones involving explicit domain restriction, such as (10e).

5.4.1 Mention-all wh-questions

Mention-all wh-questions ask for a complete specification of the individuals that have a certain property, i.e., for a specification of the complete extension of the property in the relevant domain of discourse. Under its most salient interpretation, (11) below is an example of a mention-all question.

(11) Who did Alice invite to her birthday party? \( \forall x?Pax \)

The issue expressed by (11) is resolved in a state \( s \) if the information available in \( s \) determines exactly which individuals in the domain were invited to Alice’s party, that is, if any two worlds \( w, w' \in s \) agree on the set of individuals who were invited to the party. It is easy to check that this is equivalent to the requirement that for each individual \( d \in D, s \) should determine whether or not \( d \) was invited (\( s \subseteq |Pad| \) or \( s \subseteq |\neg Pad| \)). This shows that the issue expressed by (11) is precisely the one expressed by the InqB-sentence \( \forall x?Pax \).

(12) \([11] = [\forall x?Pax] \)

\[= \{ s \mid \forall d \in D : s \subseteq |Pad| \text{ or } s \subseteq |\neg Pad| \}\]

5.4.2 Mention-some wh-questions

Mention-some wh-questions just ask for an instance of a certain property. This is exemplified in (13).

(13) What is something that Alice really likes?
The issue expressed by (13) is resolved in an information state $s$ if the information available in $s$ implies, for some object $d$, that Alice really likes $d$. This means that the issue expressed by (13) is the one expressed by the InqB-sentence $\exists x Lax$.

(14) $[(13)] = [\exists x Lax]$

$$= \{|Lad| \mid d \in D\}$$

It should be noted that the concern we mentioned above for alternative questions also arises here: in InqB, the sentence $\exists x Lax$ has non-trivial informative content, namely that there is at least one thing that Alice really likes. However, in asking the question in (13) a speaker does not present this information as new information, but rather presupposes it. Again, to capture this distinction, a presuppositional extension of InqB is required (see AnderBois, 2012; Ciardelli et al., 2012; Roelofsen, 2015a).

It is also worth remarking that, while mention-some wh-questions have received comparatively little attention in the literature (relative to mention-all wh-questions), they are extremely common in ordinary life as well as in scientific settings, as illustrated by the following examples.

(15) a. Where can I buy an Italian newspaper around here?
   b. What is a typical French dish?
   c. What is a number we can call in case of an emergency?
   d. What is an example of an arithmetic theorem that is not provable in Peano Arithmetic?

Finally, while many mention-some questions involve existential expressions (e.g., *something that Alice really likes, an Italian newspaper, a typical French dish, a number we can call*), wh-questions without such expressions can also receive mention-some interpretations, as exemplified in (16).

(16) a. What would you like to get for your birthday?
   b. Who is driving to the party tonight and has space for two extra passengers?
   c. What would hold these two sticks firmly together without damaging the paint?
   d. How do I get to the station from here?
5.4.3 Single-match wh-questions

Single-match wh-questions ask for the unique individual having a certain property. This is exemplified in (17), under the assumption that nobody can be married to more than one person.

(17) Who is Alice married to? \(\exists x \text{Max} \)

The issue expressed by (17) is resolved in an information state \(s\) if, for a specific individual \(d\) in the relevant domain of discourse, the information available in \(s\) implies that Alice is married to \(d\). This means that the issue expressed by (3) is the one that is expressed by the \(\text{InqB-sentence} \; \exists x \text{Max} \).

\[
(18) \quad [(17)] = [\exists x \text{Max}]
\]
\[
= \{|M_{\text{d}}| \mid d \in D\}
\]

As is clear from this analysis, single-match wh-questions are a special case of mention-some wh-questions in which the relevant property can be satisfied by at most one individual. It is useful to explicitly consider single-match wh-questions, since they have some special logical properties, which are not shared by other mention-some wh-questions. In particular, the alternatives for single-match wh-questions always form a partition of a subset of the logical space. In this respect, they are more similar to mention-all questions than to other mention-some questions.

5.4.4 Questions with multiple wh-phrases

Wh-questions can contain multiple wh-phrases, as exemplified in (19). Under their most salient interpretation, multiple wh-questions like (19) are mention-all wh-questions: they ask for a specification of all the individuals that stand in a certain relation—here, the relation of being married. The arity of the relation equals the number of wh-phrases in the question—in this case there are two wh-phrases, so the relation whose extension needs to be specified is a binary relation.

(19) Who is married to whom? \(\forall x \forall y \text{M}_{xy} \)

The issue expressed by (19) is resolved in an information state \(s\) in case all worlds \(w, w' \in s\) agree on the extension of the relation \(M\) (i.e., \(I_w(M) = I_{w'}(M)\)). This is equivalent to the requirement that
for every pair of individuals \(d, d' \in D\), the state \(s\) determines whether \(d\) is married to \(d'\) \((s \subseteq |M_{dd'}|)\) or \(s \subseteq |M_{dd'}|\). This shows that the issue expressed by (19) under its most salient interpretation is precisely the one expressed by the \(\text{InqB}\)-formula \(\forall x \forall y?Mxy\).

\[
\begin{align*}
(20) \quad &[(19)] = [\forall x \forall y?Mxy] \\
&= \{s \mid \forall d, d' \in D: s \subseteq |M_{dd'}| \text{ or } s \subseteq |M_{dd'}|\} \\
&= \{s \mid \forall w, w' \in s: I_w(M) = I_{w'}(M)\}
\end{align*}
\]

5.4.5 Explicit domain restriction

Sometimes, the wh-phrase in a wh-question involves an explicit domain restrictor, as exemplified in (21).

(21) Which students did Alice invite to her party?

Several proposals have been put forward in the literature concerning the contribution of the explicit restrictor in such questions. Here, we will not argue for a specific proposal, but we will consider several options and describe how these analyses can be implemented in \(\text{InqB}\).

According to Groenendijk and Stokhof (1984), (21) has two readings. Under the \textit{de dicto} reading, (21) asks for a specification of the set of students who were invited by Alice. More precisely, the issue expressed by (21) under this reading is resolved in an information state \(s\) in case for every individual \(d\), \(s\) determines whether or not \(d\) is a student who was invited by Alice. This is the issue expressed by the \(\text{InqB}\)-sentence \(\forall x?(Sx \land Pax)\).

\[
\begin{align*}
(22) \quad &[(21)]_{GS1} = [\forall x?(Sx \land Pax)] \\
&= \{s \mid \forall d \in D: s \subseteq |Sd| \cap |Pad| \text{ or } s \subseteq |Sd| \cap |Pad|\}
\end{align*}
\]

Groenendijk and Stokhof also take (21) to have a second, \textit{de re} reading, under which it asks the addressee to specify for each actual student \(d\) whether \(d\) was invited by Alice. More formally, if \(w_0\) is the actual world and \(I_{w_0}(S) = \{d_1, \ldots, d_n\}\), we have:

\[
\begin{align*}
(23) \quad &[(21)]_{GS2} = [?Pad_1 \land \cdots \land ?Pad_n] \\
&= \{s \mid \forall d \in I_{w_0}(S): s \subseteq |Pad| \text{ or } s \subseteq |Pad|\}
\end{align*}
\]

Notice that, under this reading, the issue expressed by (21) varies from world to world. Thus, it is not possible to give a single, world-independent translation in \(\text{InqB}\), where formulas express issues whose resolution conditions do not depend on the world of evaluation. Of
course, it would be possible to refine the InqB system so as to allow for such world-dependency, but we will not pursue such a refinement in detail here.

Velissaratou (2000) puts forward a different analysis of which-questions. In her theory, (21) expresses an issue which is resolved if for every individual $d$, under the assumption that $d$ is a student, it is known whether $d$ was invited by Alice. More formally, an information state $s$ counts as settling the relevant issue if for every $d$, the state $s \cap |Sd|$ which results from assuming that $d$ is a student settles whether or not $d$ was invited by Alice (that is, $s \cap |Sd| \subseteq |Pad|$ or $s \cap |Sd| \subseteq \overline{|Pad|}$). It is easy to check that this is precisely the issue expressed by the InqB-sentence $\forall x(Sx \rightarrow ?Pax)$.

\[ (24) \quad [(21)]_V = [\forall x(Sx \rightarrow ?Pax)] = \{ s | \forall d \in D : s \cap |Sd| \subseteq |Pad| \text{ or } s \cap |Sd| \subseteq \overline{|Pad|} \} \]

### 5.5 Question coordination and conditionalization

As we mentioned in Section 1.1.3, questions, just like statements, can be coordinated by means of conjunction and disjunction, and conditionalized by means of if-clauses. We will see that in each case, the relevant operation is matched by the corresponding logical operation in InqB.

#### 5.5.1 Conjoined questions

A question like (25), which consists of two polar questions coordinated by means of the conjunction word and, asks for information which resolves both of the conjuncts.

(25) Does Alice like Bob, and does he like her? $\Lab \land \Lba$

That is, the issue expressed by (25) is resolved in an information state $s$ just in case $s$ resolves both the issue whether Alice likes Bob, and the issue whether Bob likes Alice. This is precisely the issue expressed by the conjunction $\Lab \land \Lba$ in InqB.

\[ (26) \quad [(25)] = [\Lab \land \Lba] = [\Lab] \cap [\Lba] = \{ |\Lab| \cap |\Lba|, |\Lab| \cap \overline{|\Lba|}, \overline{|\Lab|} \cap |\Lba|, \overline{|\Lab|} \cap \overline{|\Lba|} \} \]
Notice that the English conjunction word *and* can simply be translated here as the logical connective $\land$. This holds in general: given two questions $Q$ and $Q'$ whose InqB-translations are $\mu$ and $\mu'$, the conjunctive question $Q$ and $Q'$ can be translated as $\mu \land \mu'$.

\[(Q \text{ and } Q') = \{s \mid s \in [Q] \text{ and } s \in [Q']\} = [Q] \cap [Q'] = [\mu] \cap [\mu'] = [\mu \land \mu']\]

### 5.5.2 Disjoined questions

A question like (28), consisting of two mention-some wh-questions which are coordinated by means of the disjunction word *or*, asks for information which resolves either one of the disjuncts.$^3$

(28) Who can drive Alice to the party, or who can lend her a car? $\exists x Dxa \lor \exists x Lxa$

More precisely, the issue expressed by (28) is resolved in an information state $s$ if $s$ implies for some individual $d$ that $d$ can drive Alice to the party, or if $s$ implies for some individual $d$ that $d$ can lend Alice a car. If the disjuncts of (28) are translated as $\exists x Dxa$ and $\exists x Lxa$, respectively, then the issue expressed by (28) is precisely the issue expressed by the disjunction $\exists x Dxa \lor \exists x Lxa$.

\[(29) \quad [(28)] = [\exists x Dxa \lor \exists x Lxa] = [\exists x Dxa] \cup [\exists x Lxa] = \{|Dda| \mid d \in D\} \uparrow \cup \{|Lda| \mid d \in D\} \uparrow\]

Notice that, as in the case of conjunction, the English disjunction word *or* can simply be translated here as the logical connective $\lor$.

### 5.5.3 Conditional questions

Conditional questions ask for a resolution of a question, specified by the main clause, under a certain assumption which is specified by an adjoined *if*-clause. As an example, consider (30), which is obtained by conditionalizing a single-match wh-question.

---

$^3$ We should note here, as we also did in footnote 3 in Section 1.1.3, that disjunctions of questions are much less common in language than conjunctions of questions. Some authors have even claimed that questions cannot be disjoined at all (Szabolcsi, 1997; Krifka, 2001b). We are convinced by examples like (28) that disjoining questions is in principle possible. This point will be discussed in more detail in Section 9.2.2.
(30) If Alice wins two tickets, who will she take with her?

\[ Wa \rightarrow \exists x Tax \]

The issue expressed by (30) is resolved in an information state \( s \) if restricting \( s \) to those worlds where Alice wins two tickets results in a state \( s \cap |Wa| \) which resolves the issue of who Alice will take with her, i.e., a state which implies for some individual \( d \) that Alice will take \( d \) with her.

\[ (30) \quad [ (30) ] = [ Wa \rightarrow \exists x Tax ] \\
= \{ s \mid s \cap |Wa| \in [ \exists x Tax ] \} \\
= \{ s \mid \text{for some } d \in D : s \cap |Wa| \subseteq |Tad| \} \]

Notice that the \( \text{InqB} \)-translation of (30) is a conditional whose antecedent is the translation of the \( \text{if} \)-clause, and whose consequent is the translation of the main clause. This is not a coincidence, but a result that holds generally for indicative conditional questions. Suppose that \( A \) is a statement whose \( \text{InqB} \)-translation is a non-inquisitive formula \( \alpha \), and suppose \( Q \) is a question whose \( \text{InqB} \)-translation is \( \mu \). The conditional question \( \text{if } A, Q \) is resolved in an information state \( s \) if restricting \( s \) to those worlds where \( A \) is true results in a state which resolves \( Q \). Using the fact that \( \alpha \) is non-inquisitive, we have:

\[ (32) \quad [ \text{if } A, Q ] = \{ s \mid s \cap |A| \in [ Q ] \} \\
= \{ s \mid s \cap |\alpha| \in [ \mu ] \} \\
= \{ s \mid \forall t \subseteq s : \text{if } t \in [ \alpha ] \text{ then } t \in [ \mu ] \} \\
= [ \alpha \rightarrow \mu ] \]

This ensures that \( \text{if } A, Q \) can be translated as \( \alpha \rightarrow \mu \).

Besides indicative conditional questions, there are also counterfactual conditional questions, such as (33).

(33) If Alice had won two tickets, who would she have taken with her?

In order to translate (33), we need to extend \( \text{InqB} \) with an analysis of counterfactual conditionals. We will come back to this issue in Chapter 7.

5.6 Limitations and extensions

While we have illustrated above that \( \text{InqB} \) allows us to formally capture the resolution conditions of many kinds of questions occurring in natural language, there are also some aspects of the interpretation of questions that are beyond the immediate scope of this basic framework.
and require suitable extensions. For instance, we already remarked in Sections 5.2 and 5.4 that, besides requesting information, questions may also presuppose certain information, and \textit{InqB} as such is not equipped to encode such presuppositions. Below we will briefly discuss two other aspects of question meaning that are beyond the immediate reach of \textit{InqB}, with pointers to the literature for further discussion of the required extensions.

5.6.1 Beyond resolution conditions: anaphora and bias

Compare the prototypical polar question in (34) with the somewhat less prototypical questions in (35) and (36).

(34) Is the door open?
(35) Is the door open or closed?
(36) The door is open, isn’t it?

An information state resolves the issue expressed by any of these questions if and only if it either implies that the door is open, or that the door is closed. Thus, these questions have exactly the same resolution conditions, and therefore they express exactly the same issue. This commonality is captured in \textit{InqB}: (34), (35), and (36) all express the same proposition containing two alternatives, one consisting of all worlds where the door is open, and one consisting of all worlds where the door is closed.

However, despite the fact that the questions in (34)–(36) have exactly the same resolution conditions, they clearly differ in their overall conversational effects. For instance, (34) allows for \textit{yes/no} answers and other anaphoric continuations, while (35) does not.

(37) A: Is the door open?
    B: Yes. / No. / I think so.

(38) A: Is the door open or closed?
    B: \#Yes. / \#No. / \#I think so.

Moreover, while (34) can be felicitously uttered in a situation in which the speaker expects the door to be closed, the tag-question in (36) cannot. In other words, the latter conveys a bias on the part of the speaker that the door is open.

(39) Is the door open? I expect that it isn’t.
(40) The door is open, isn’t it? \#I expect that it isn’t.
5.6.2 Contextual parameters

Just like the information provided by a natural language statement, so also the issue expressed by a natural language question is rarely completely determined by grammar alone; rather, it depends on the conversational context in various ways. Some of the relevant contextual factors can be illustrated by considering the following examples.

(41)  a. Which students passed the exam?
    b. What is the winning card?
    c. Who is driving to the party tonight?
    d. Where is Mary?

A first important contextual parameter is the intended domain of quantification. For instance, the issue expressed by (41a) depends on the set of students which are relevant in a particular context.

A second contextual parameter manifests itself in (41b). The issue expressed by this question does not only depend on the intended
domain of quantification, but also on the intended method of identification (Aloni, 2005). Suppose that the question is asked in a situation in which there are two cards on the table, face down. If (41b) is asked by someone who wants to pick the winning card, it is resolved by any piece of information that conveys whether the winning card is the one on the left or the one on the right. On the other hand, if (41b) is asked by someone who does not know the rules of the game and wants a description of the winning card in terms of suit and number, then it is resolved by a piece of information that conveys, e.g., that the winning card is the six of hearts.

The issue expressed by (41c) depends, again besides the intended domain of quantification, also on the kind of goal that the questioner is trying to achieve in asking the question (van Rooij, 2003). For instance, she may be trying to identify someone who could give her a ride to the party, but she may also want to draw up a list of people driving to the party. In the first case, the question gets a mention-some interpretation: to resolve it, it suffices to specify one person who is driving to the party. In the second case, the question gets a mention-all interpretation: in this case, to resolve the question it is necessary to specify the complete set of people who are driving to the party.

Finally, the issue expressed by (41d) depends on the intended level of granularity (Ginzburg, 1995). In some contexts, the information provided by (42a) is sufficient to resolve the question. In other contexts, Mary’s location needs to be determined more precisely, for instance by providing the information in (42b).

(42)   a. Mary is at home.
   b. Mary is in the bathroom.

Some sources of context-dependency are already taken into account in InqB. After all, an InqB-sentence expresses different issues in different information models. In this way, InqB captures the way in which the issue expressed by a question depends on the intended domain of quantification $D$, and on the set of worlds $W$ which are considered possible in the given context.

On the other hand, some extra machinery would have to be added to InqB to model the influence of other contextual factors, such as the method of identification, the questioner’s goals, and the intended degree of granularity. In principle, it seems that the existing techniques
designed to deal with these contextual factors (Ginzburg, 1995; van Rooij, 2003; Aloni, 2005) can be combined with an inquisitive approach to question semantics. However, a concrete implementation of these techniques in the inquisitive setting has not been pursued yet.

5.7 Exercises

Exercise 5.1 Disjunctive questions

Consider the following disjunctive question, in the given context:

(43) Context: Mary keeps a small collection of things that could be nice gifts. At the moment she has two items in her collection: a small Picasso reproduction and a beautiful handicraft book rest that she got on a trip to India. A good friend of hers, Sue, is getting married. She knows that Sue loves ceramics, so that’s what she’ll give her. But she also wants to give her fiancé Ben something. She doesn’t know him so well yet, though, so she asks Sue:

Does Ben like Picasso↑ or does he read a lot↑?

Consider the following possible translations in InqB:

(44) a. Pb ∨ Rb
b. ?(Pb ∨ Rb)
c. !(Pb ∨ Rb)
d. ?Pb ∨ ?Rb

1. What are the resolution conditions of the question according to these translations?

2. Which translation is the most appropriate? Why?

Exercise 5.2 Quantifying into questions

Consider the following question, with two possible translations into InqB:

(45) What did every man eat?

a. ∀y∀x(Mx → Exy) [narrow scope for every man]
b. ∀x(Mx → ∀yExy) [wide scope for every man]

1. According to these two translations, what are the resolution conditions of the question?

2. Do the two translations indeed correspond to two possible interpretations of the question?
Exercise 5.3 Which questions

Consider the which question in (46), the polar question in (47), and the statements in (48)[a–d], with the corresponding InqB-translations given on the right.

(46) Which spies were arrested?

(47) Were any spies arrested?  ?∃x(Sx ∧ Ax)

(48) a. No spies were arrested.  ∀x(Sx → ¬Ax)
    b. All spies were arrested.  ∀x(Sx → Ax)
    c. Alice is the only spy who was arrested.  ∀x((Sx ∧ Ax) ↔ x = a)
    d. Either Alice is the only spy who was arrested,
       or Alice is not a spy and no spies were arrested.
       ∀x((Sx ∧ Ax) ↔ x = a) ∨ (¬Sa ∧ ∀x¬(Sx ∧ Ax))

Recall that Groenendijk and Stokhof analyse (46) (on the de dicto reading) as ∀x?(Sx ∧ Ax), while Velissaratou (2000) analyses (46) as ∀x(Sx → ?Ax).

1. Which of the statements in (48) resolve (46) according to these analyses?
2. According to these analyses, does (46) entail (47)?
3. Discuss how well these predictions match your linguistic intuitions.
6

Disjunction, clause typing, and intonation

In Chapter 5 we considered various kinds of questions in English. We described their resolution conditions and specified their translations in InqB. We saw in particular that the interpretation of questions containing disjunction heavily depends on intonation. In the present chapter we will have a closer look at how disjunction interacts with clause typing (declarative versus interrogative) and intonation (intonational phrase structure and final pitch contours), showing that inquisitive semantics allows us to treat disjunction uniformly across statements and questions with various intonation patterns.

The types of sentences that we will be primarily interested in are exemplified in (1)–(5) below—though we will see shortly that our analysis applies to some closely related sentence types as well. As before, we use arrows to indicate falling and rising intonation at the end of an intonational phrase. Moreover, if two disjuncts are pronounced within a single intonational phrase, we use hyphens to explicitly indicate the absence of an intonational phrase break between the two disjuncts.

(1) Does Igor speak Spanish-or-French↑?
(2) Does Igor speak Spanish↑ or does he speak French↓?
(3) Does Igor speak Spanish↑ or does he speak French↑?
(4) Igor speaks Spanish-or-French↑.
(5) Igor speaks Spanish↑ or he speaks French↓.

The sentence types exemplified in (1)–(3) were already discussed in the previous chapter: (1) is a polar disjunctive question, which raises an issue whose resolution requires establishing that Igor speaks either Spanish or French, or establishing that he doesn’t speak either of the two languages; (2) is an alternative question, which presupposes that Igor speaks either Spanish or French and raises an issue whose resolution...
requires establishing which of the two languages he speaks; and finally, (3) is an open disjunctive question, which raises an issue that can be resolved in three ways: by establishing that Igor speaks Spanish, by establishing that he speaks French, or by establishing that he does not speak either of the two languages.

The disjunctive sentence types in (4) and (5) were not discussed in the previous chapter yet, because they are statements rather than questions. The difference between them is that in (4) the two disjuncts are part of a single clause, while in (5) they are separate clauses. Unlike in the case of questions, however, this difference in syntactic structure does not result in a difference in interpretation. Both (4) and (5) convey the information that Igor speaks Spanish or French, and do not request any further information.

The similarities and differences in interpretation between (1)–(5) should be derivable in a systematic way from the similarities and differences in form between these sentences. Note that there are three formal aspects that seem to play a particularly important role in determining the interpretation of (1)–(5).

The first important aspect is clause type marking: the clauses in (1)–(3) are marked as interrogative clauses by the presence of a fronted auxiliary verb, while the clauses in (4)–(5) are marked as declarative clauses by the absence of such fronted auxiliary verbs. This has a semantic effect, as can be seen by comparing (2) and (5). We assume that these two examples form a minimal pair, i.e., that they only differ in that the former involves interrogative clause type marking, while the latter involves declarative clause type marking. This, then, should be the source of the difference in interpretation between the two sentences.

As we saw in the previous chapter, open disjunctive questions are only used in rather specific kinds of contexts. For the open disjunctive question in (3) one could, for instance, imagine the following context. Igor has electronically applied for a grant from the European Research Council. Two officers are processing the applications together, and one of them is starting to compose a response letter to Igor. By default, such letters are in English, but if the applicant has indicated in their application form that they prefer correspondence in Spanish or French, then the letter will be in that language. The officer who is starting to compose the letter to Igor does not remember which language he had indicated on his application form. The other officer still has Igor's application form on his computer screen. In this situation, it is natural for the first officer to ask his colleague the open disjunctive question in (3). On the other hand, the alternative question in (2) would not be appropriate, since the officer does not want to presuppose that Igor had indicated a preference for Spanish or French on his form, and the polar disjunctive question in (1) would not be suitable either, because the issue raised by that question could be resolved just by establishing that Igor speaks either Spanish or French, without establishing which of the two; this would not be sufficient to decide in which language the letter should be composed.
A second formal aspect that matters is whether the final pitch contour is falling or rising. That this has a semantic effect can be seen by comparing (2) and (3). Again, we take this to be a minimal pair—the only difference here is that (2) involves a final fall while (3) involves a final rise. This, then, must be the source of the difference in interpretation between the two.

Finally, a third important aspect is syntactic structure, in particular whether the two disjuncts are part of a single clause, or rather form two separate clauses. The fact that this has a semantic effect can be seen by comparing (1) and (3). Again, we take this to be a minimal pair. The only difference is that in (1) the two disjuncts are part of a single clause, while in (3) they form two separate clauses. This must be the source of the difference in interpretation between the two.\(^2\)

Thus, our aim will be to show how the differences in meaning between (1)–(5) may be derived from the differences in clause type marking, final pitch contour, and syntactic structure. In doing so, we will maintain a uniform treatment of the English disjunction word or as expressing the join operation, just like the InqB connective \(\lor\). In this respect, our account will differ from many previous analyses—in particular, the classical theories of Karttunen (2011) and Groenendijk and Stokhof (1984)—which do not offer a uniform treatment of disjunction across all types of disjunctive questions and statements, but rather assume that the semantic contribution of disjunction in alternative questions is different from its contribution in polar disjunctive questions and in statements.\(^3\)

\(^2\) One may wonder whether sentences like (1) could possibly also be treated as cases where disjunction applies to two full clauses, where the second clause is almost entirely elided, i.e., left unpronounced. This, however, would be incompatible with the commonplace assumption that every syntactic clause boundary must align with an intonational phrase boundary (see, e.g., Truckenbrodt, 2007; Selkirk, 2011). If the sentence in (1) consisted of two full clauses, then there would have to be an intonational phrase break after the first, i.e., immediately preceding the disjunction word. Since by assumption there are no such intonational phrase breaks, (1) really has to be treated as involving a single clause, containing a sub-clausal disjunction.

Note that there are also cases like (i) below, which are just like (1) except that they do exhibit an intonational phrase boundary after the first disjunct:

(i) Does Igor speak Spanish\(^1\) or French?\(^1\)?

We will leave such cases out of consideration here and concentrate on those where it is clear whether disjunction applies to two separate clauses or clause-internally.

\(^3\) The uniform treatment of disjunction across questions and statements to be presented here is closely related to the treatment of disjunction in alternative semantics (Simons, 2005; Alonso-Ovalle, 2006; Aloni, 2007). See Section 4.8 for some discussion of how the latter treatment of disjunction is related to the inquisitive one. For more elaborate comparison, see Roelofsen (2015b); Ciardelli et al. (2017a); Ciardelli and Roelofsen (2018).
The account presented here will be a simplified version of the one developed in Roelofsen (2013c, 2015a). The same simplified account has also been presented in Roelofsen and Farkas (2015), where it serves as the basis for a theory of answer particles like yes and no. The main reason we present only a simplified version of the account here is that the full account does not only aim to capture the informative and inquisitive content of the various sentence types, but also their presuppositional content, which, as discussed in Section 5.2, requires an extension of the InqB system. While such an extension increases the empirical coverage of the account, a simplified non-presuppositional version should suffice to demonstrate the advantages of inquisitive semantics in formulating a uniform account of disjunction, clause type marking, and the relevant intonational features.4

While our focus here will be on English, we expect that the basic semantic operations that our account associates with the relevant lexical, morphological, and intonational features may play a central role in the interpretation of similar constructions in other languages as well. The division of labor between the various elements is bound to vary from language to language, but we expect that the basic repertoire of semantic operations that our account draws on will be relatively stable across languages.

We will proceed as follows. Section 6.1 provides an informal characterization of the kind of syntactic structures that we take sentences like (1)–(5) to have, Section 6.2 formally specifies their logical forms, and Section 6.3 specifies how these logical forms are to be interpreted in InqB.

6.1 List structures

Drawing inspiration from Zimmermann (2000), we will view the sentence types exemplified in (1)–(5) as lists.5 Lists either consist of a single

4 Besides leaving presuppositions out of consideration, the present account is simplified in another respect as well. Namely, in order to derive the fact that alternative questions presuppose that exactly one of the disjuncts holds, the full account assumes that such questions involve an exclusive strengthening operator. For simplicity, this operator is left out of consideration here.

5 The idea that disjunction can be used to form lists has also been put forth by Simons (2001, p. 616), independently of Zimmermann (2000). In Simons' work, however, this idea does not form the basis for a particular semantic treatment of disjunctive sentences and their various intonational features, but is rather part of a pragmatic explanation for the fact that disjunctive declaratives are typically much more natural in response to a given question.
clause, as in (1) and (4), or of multiple clauses separated by disjunction, as in (2), (3), and (5). We will refer to these clauses as 'list items'. We think of lists as being either open (signaled by a final rise), as in (1) and (3), or closed (signaled by a final fall), as in (2), (4), and (5). Non-final list items are canonically pronounced with rising intonation, both in open and in closed lists. Moreover, each item is pronounced in a separate intonational phrase, which means that there is an intonational phrase break after each non-final item, before the disjunction word. In fact, two non-final list items may be separated just by an intonational phrase break, i.e., the disjunction word may be omitted if neither of the items is final.

Thus, we take lists to differ along three basic parameters: they can be declarative or interrogative, they can be open or closed, and they can consist of a single clause or of multiple clauses separated by disjunction. Given these parameters, there are in total $2 \times 2 \times 2 = 8$ types of lists, five of which were exemplified in (1)–(5): the polar question in (1) is a mono-clausal open interrogative list, the alternative question in (2) a multi-clausal closed interrogative list, the open disjunctive question in (3) a multi-clausal open interrogative list, the statement in (4) a mono-clausal closed declarative list, and the statement in (5) a multi-clausal closed declarative list.

Note that, while the mono-clausal lists in (1) and (4) contain a disjunction, this is not a necessary feature of mono-clausal lists. Thus, plain non-disjunctive polar questions and statements, exemplified in (6)–(7) below, also count as lists under our perspective, and should therefore also be covered by our account.

(6) *Non-disjunctive mono-clausal open interrogative:*

Does Igor speak Spanish↑?

(7) *Non-disjunctive mono-clausal closed declarative:*

Igor speaks Spanish↓.

So far, only five out of eight list types have been exemplified. The three remaining list types are exemplified in (8)–(10) below.

than truth-conditionally equivalent non-disjunctive sentences. For instance, if the question is why Jane is not picking up the phone, then (ia) is a much more natural answer than (ib).

(i) a. Either she isn’t home, or she can’t hear the phone.

   b. It’s not the case that she is at home and she can hear the phone.

To the extent that Simons’ analysis of this phenomenon is successful, it provides independent motivation for our general outlook on disjunctive sentences as lists. A proper discussion of Simons’ analysis, however, is beyond the scope of this book.
(8) Mono-clausal open declarative:
    Igor speaks Spanish↑?

(9) Mono-clausal closed interrogative:
    Does Igor speak Spanish↑?

(10) Multi-clausal open declarative:
    Igor speaks Spanish↑ or he speaks French↑?

Sentences like (8), with declarative clause type marking and rising intonation, are referred to in the literature as *rising declaratives* or *declarative questions* (see, e.g., Gunlogson, 2001, 2008; Malamud and Stephenson, 2015; Farkas and Roelofsen, 2017; Westera, 2017). They raise the same issue as the corresponding rising polar interrogative. In the case of (8), this issue can be resolved either by establishing that Igor speaks Spanish or by establishing that he does not. However, unlike rising polar interrogatives, rising declaratives also convey some sort of *bias* towards the alternative that is explicitly mentioned, here the one that Igor does speak Spanish. We will not present an account of this bias here, but will derive that a rising declarative expresses the same issue as the corresponding rising polar interrogative, and offer an explanation of the fact that among a rising declarative and a rising polar interrogative expressing the same issue, the rising polar interrogative is seen as the canonical form to express that issue, and the rising declarative as a more ‘marked’ form.6 In light of the general tendency for the overall communicative effect of marked forms to be more complex than that of their unmarked, canonical counterparts (see, e.g., Horn, 1984; Blutner, 2000), it is unsurprising that rising declaratives have a special discourse effect (signalling a bias), which plain polar questions lack. For an extension of the account to be presented here which discusses the bias conveyed by rising declaratives and other marked question types in detail, we refer to Farkas and Roelofsen (2017).

Returning to the different list types, in (9) we have a mono-clausal closed interrogative list, i.e., a falling polar interrogative. This sentence, again, expresses exactly the same issue as the corresponding rising polar interrogative, and is also generally seen as a more marked form than the latter.7 Whether falling polar interrogatives are also systematically

6 For instance, Quirk et al. (1985) state that “Yes–no questions are usually formed by placing the operator before the subject and giving the sentence a rising intonation” (p. 807).

7 For instance, Hedberg et al. (2014) state that “the low-rise nuclear contour (e.g., L↑H-H%) is the unmarked question contour and is by far the most frequently occurring. Yes-no questions with falling intonation (e.g. H↑L-L%) do not occur frequently, but
associated with a certain special discourse effect, like rising declaratives, is not so clear—certainly, there does not seem to be a broad consensus in the literature on what this effect would be (see Hedberg et al., 2014, for relevant recent discussion). In any case, our aim will just be to account for the fact that a falling polar interrogative raises the same issue as the corresponding rising polar interrogative, and to offer an explanation for the fact that it is seen as a relatively marked sentence type.

Finally, in (10) we have a multi-clausal open declarative list, i.e., a sentence consisting of two rising declarative clauses, joined by disjunction. This sentence type strikes us as very odd. It is difficult, if not impossible, to imagine a context in which it could be felicitously used. As far as we know, it has not been discussed in any depth in the literature. It is an interesting fact, however, that the commonplace ingredients that make up this construction—two declarative clauses, disjunction, and rising intonation—cannot be combined in this particular way. We will not be able to give an account of this empirical observation here; this would require a more detailed analysis of the bias associated with mono-clausal rising declaratives, as pursued in Farkas and Roelofsen (2017). However, we will offer an explanation of the fact that the construction is marked, in the sense that it is not the canonical way of conveying the issue that it expresses.

6.2 Logical forms

We now turn to a more formal specification of the syntactic structures—the logical forms—that we take list structures to have. Globally, we assume that a list with \( n \) items has the following logical form:

\[
\begin{array}{c}
\text{OPEN/CLOSED} \\
\text{DECL/INT} \\
\text{item}_1 \\
\text{or} \\
\text{...} \\
\text{or} \\
\text{item}_n
\end{array}
\]

when they do, they can be classified in speech act terms as 'non-genuine' questions, where one or more felicity conditions on genuine questions are not met.
We will refer to open/closed as *completion markers*, to decl/int as *complementizers*, and to the rest of the structure as the *body* of the list. We assume that each item in the body of the list is a full clause, headed by a declarative or interrogative *clause type marker*, C_{DECL} or C_{INT}, depending on whether the list as a whole is headed by decl or int, respectively. That is, if the complementizer of a list is decl, then all clauses in the body of that list must be headed by C_{DECL}, and similarly if the complementizer of the list is int.

To give a concrete example, the polar question in (1), which consists of a single clause containing a disjunctive phrase and exhibits a final rise, is taken to have the following structure:

\[(12)\]

\[
\text{OPEN}\quad \text{INT}\quad \text{item}_1 \\
C_{\text{INT}} \text{ does Igor speak Spanish or French}
\]

On the other hand, the alternative question in (2), which consists of two clauses and exhibits a final fall, is taken to have the following structure:

\[(13)\]

\[
\text{CLOSED}\quad \text{INT}\quad \text{item}_1 \\
C_{\text{INT}} \text{ does Igor speak Spanish or item}_2 \quad C_{\text{INT}} \text{ does he speak French}
\]

### 6.3 Interpreting logical forms

We will now specify a semantic interpretation of these logical forms by translating them into InqB. Thereby we associate each logical form with a proposition, namely the proposition expressed by the formula that serves as its translation.
The body of a list

Let us first consider the body of a list, and after that turn to complementizers and completion markers. Recall that the body of a list consists of one or more list items, separated by disjunction. Every list item, in turn, is a full clause headed by a declarative or interrogative clause type marker ($C_{\text{DECL/INT}}$). The rest of the clause is a tense phrase (TP), which may itself contain a disjunction.

The translation procedure is very straightforward. Any disjunction is translated as $\lor$, no matter whether it separates two list items or occurs within one of the list items. Every clause type marker, be it declarative or interrogative, is translated as $!$. The rationale for this is that every list item is seen, intuitively speaking, as one block, i.e., as contributing a single alternative to the proposition expressed by the list as a whole. This is ensured by applying $!$, which turns any proposition $P$ into a proposition with a single alternative, $\bigcup P$. Otherwise the procedure is straightforward: basic clauses are translated as atomic formulas, and English conjunction, disjunction, and negation are translated as the corresponding $\text{InqB}$ connectives. Thus, the body of a list is translated according to the rule in (14), where $\varphi_1, \ldots, \varphi_n$ are standard translations of TP$_1, \ldots, $ TP$_n$ into our logical language.

(14) Rule for translating the body of a list:

$$\left[ [C_{\text{DECL/INT}} \text{TP}_1] \lor \cdots \lor [C_{\text{DECL/INT}} \text{TP}_n] \right] \rightarrow !\varphi_1 \lor \cdots \lor !\varphi_n$$

Returning to our concrete examples above, if we translate *Igor speaks Spanish* as $p$ and *Igor speaks French* as $q$, then we get the following translations for the list bodies of (1) and (2), respectively.$^9$

$^9$ In previous chapters, the logical language that we assumed was a first-order language, with predicate symbols and individual constants and variables. Here, we simply use a propositional language with atomic proposition symbols $p$ and $q$, since the internal structure of predicate-argument combinations is irrelevant for present purposes.
disjunction, clause typing, and intonation

(15) \[ C_{\text{INT}} \text{ does Igor speak Spanish or French} \] \[ \leadsto ! (p \lor q) \]

(16) \[ C_{\text{INT}} \text{ does Igor speak Spanish} \text{ or } C_{\text{INT}} \text{ does he speak French} \] \[ \leadsto !p \lor !q \]

Complementizers and completion markers  Now let us turn to complementizers and completion markers. To specify their semantic contribution it is convenient to use some notation and terminology from type theory.\(^{10}\) Recall that in inquisitive semantics, propositions are sets of sets of possible worlds, i.e., objects of type \(\langle \langle s, t \rangle, t \rangle \). Let us abbreviate this type as \(T\). Now, we will treat \(\text{decl} \) and \(\text{int} \) as propositional operators, i.e., as functions that take a proposition as their input, and deliver another proposition as their output. This means that \(\text{decl} \) and \(\text{int} \) are of type \(\langle T, T \rangle \). On the other hand, we will treat \(\text{open} \) and \(\text{closed} \) as modifiers of propositional operators, i.e., as functions that take a propositional operator as their input, and deliver a modified propositional operator as their output. So \(\text{open} \) and \(\text{closed} \) are of type \(\langle \langle T, T \rangle, \langle T, T \rangle \rangle \). It will become clear in a moment why \(\text{open} \) and \(\text{closed} \) are treated as having this somewhat more complex type, rather than simply \(\langle T, T \rangle \), like \(\text{decl} \) and \(\text{int} \). We will now take a more detailed look at each of the complementizers and completion markers in turn.

Let us start with \(\text{decl} \). We will treat \(\text{decl} \) as making a list purely informative, i.e., as \textit{eliminating inquisitiveness}. This effect can be achieved straightforwardly by treating \(\text{decl} \) as a function that takes the proposition \(P \) expressed by the body of a list as its input and applies the projection operator \(! \) to it, returning \(!P \). Using type-theoretic notation, this can be formulated concisely as follows:

\[ (17) \quad \text{decl} \quad \leadsto \lambda P. !P \]

Next, consider \(\text{int} \). We take the role of this operator to be that of \textit{ensuring inquisitiveness}. This is done by applying a conditional variant of the \(? \) operator, which we will denote here as \(? \). If the proposition \(P \) that \(? \) takes as its input is not yet inquisitive, then \(? \) is applied to it. On the other hand, if \(P \) is already inquisitive, then it is left untouched. The only

\(^{10}\) We will only use some type-theoretical notation here in the meta-language to describe functions (as in, e.g., Heim and Kratzer, 1998). A more rigorous approach would be to extend the \(\text{InqB} \) system to a full-fledged type theoretic framework (as done in Ciardelli, Roelofsen, and Theiler, 2017a). We leave this step implicit here because it would involve quite some technicalities which are to a large extent orthogonal to our present concerns.
case in which this procedure does not yield an inquisitive output is when $P$ is a tautology or a contradiction. In this case $\langle ? \rangle P$ is a tautology. In all other cases, $\langle ? \rangle P$ is inquisitive. Thus, we assume the following treatment of \textsc{int}:

\begin{equation}
\text{int} \quad \leadsto \quad \lambda P. \langle ? \rangle P
\end{equation}

Finally, let us consider \textsc{open} and \textsc{closed}. Intuitively speaking, we treat these completion markers as encoding whether the list is ‘complete’ and ready to be ‘sealed off’, or rather left ‘open-ended’. The role of \textsc{closed} is to mark the list as being complete, and to allow \textsc{decl} or \textsc{int}, whichever is present, to seal off the list. Thus, \textsc{closed} is simply treated as the identity function, leaving the propositional operator $\pi$ expressed by \textsc{decl} or \textsc{int} untouched and letting it apply to the proposition expressed by the body of the list.

\begin{equation}
\text{closed} \quad \leadsto \quad \lambda \pi. \pi
\end{equation}

On the other hand, the role of \textsc{open} is to mark the list as being open-ended. It prevents \textsc{decl}/\textsc{int} from sealing off the body of the list, and instead applies the $\pi$ operator, which adds the set-theoretic complement of $\bigcup P$ as an additional alternative. This captures what we take to be the characteristic semantic property of open lists, which is that they always leave open the possibility that none of the given list items holds. Thus, unlike \textsc{closed}, \textsc{open} prevents the operator $\pi$ expressed by \textsc{decl} or \textsc{int} from becoming operative, and instead applies $\pi$ to the proposition $P$ expressed by the body of the list.

\begin{equation}
\text{open} \quad \leadsto \quad \lambda \pi. \lambda P. \pi P
\end{equation}

In total there are four types of lists, each featuring a combination of one complementizer and one completion marker. From the treatment of the individual complementizers and completion markers given above, it follows that the four types of lists are translated into our logical language as specified in (21) below, where in each case $\varphi$ stands for the translation of the body of the list, obtained according to the rule in (14) above.

\footnote{In addition to ensuring inquisitiveness, Roelofsen (2013c, 2015a) assumes that \textsc{int} has another effect as well: it ensures non-informativity, by introducing a presupposition that the actual world must be contained in $\bigcup P$. This second aspect of interrogativity is important in order to account for the presuppositional component of alternative questions (discussed in Section 5.2). Since in this section we are casting our account in \textsc{InqB}, which does not represent presuppositions, we set aside this second effect of \textsc{int}.}
Table 6.1 Representative examples of all types of lists considered.

<table>
<thead>
<tr>
<th>Translation</th>
<th>Simplified</th>
</tr>
</thead>
<tbody>
<tr>
<td>—Closed declaratives—</td>
<td></td>
</tr>
<tr>
<td>Igor speaks Spanish↓</td>
<td>![p] p</td>
</tr>
<tr>
<td>Igor speaks Spanish-or-French↓</td>
<td>![p ∨ q] ![p ∨ q]</td>
</tr>
<tr>
<td>Igor speaks Spanish↑ or he speaks French↑</td>
<td>![p ∨ !q] ![p ∨ q]</td>
</tr>
<tr>
<td>—Open declaratives—</td>
<td></td>
</tr>
<tr>
<td>Does Igor speak Spanish↑?</td>
<td>![p] ![p]</td>
</tr>
<tr>
<td>Does Igor speak Spanish-or-French↑?</td>
<td>![p ∨ q] ![p ∨ q]</td>
</tr>
<tr>
<td>Does Igor speak Spanish↑ or does he speak French↑?</td>
<td>![p ∨ !q] ![p ∨ q]</td>
</tr>
<tr>
<td>—Closed interrogatives—</td>
<td></td>
</tr>
<tr>
<td>Does Igor speak Spanish↑?</td>
<td>![p] ![p]</td>
</tr>
<tr>
<td>Does Igor speak Spanish-or-French↑?</td>
<td>![p ∨ q] ![p ∨ q]</td>
</tr>
<tr>
<td>Does Igor speak Spanish↑ or does he speak French↑?</td>
<td>![p ∨ !q] ![p ∨ q]</td>
</tr>
<tr>
<td>—Open interrogatives—</td>
<td></td>
</tr>
<tr>
<td>Does Igor speak Spanish↑?</td>
<td>![p] ![p]</td>
</tr>
<tr>
<td>Does Igor speak Spanish-or-French↑?</td>
<td>![p ∨ q] ![p ∨ q]</td>
</tr>
<tr>
<td>Does Igor speak Spanish↑ or does he speak French↑?</td>
<td>![p ∨ !q] ![p ∨ q]</td>
</tr>
</tbody>
</table>

(21) Rules for translating lists

a. [[CLOSED DECL] body] \leadsto ![φ]
b. [[CLOSED INT] body] \leadsto ⟨?⟩φ
c. [[OPEN DECL] body] \leadsto ?φ
d. [[OPEN INT] body] \leadsto ?φ

The rules in (14) and (21) together give a complete specification of how to translate declarative and interrogative lists in English into our logical language. In Table 6.1 we provide translations for a number of examples that are representative for all the types of lists that we are concerned with. In the Table, as well as in the discussion below, we use hyphens (Spanish-or-French) to indicate that the two disjuncts are pronounced within one intonational phrase. In the translations of the examples, \( p \) stands for Igor speaks Spanish and \( q \) for Igor speaks French, as above. In each case we provide the direct translation and also a simpler formula that is semantically equivalent in \( \text{InqB} \) to the direct translation. The propositions expressed by all these simplified translations are depicted in Figure 6.1.

The following two sections discuss these results in more detail. First, in Section 6.4, we consider the ‘unmarked’ disjunctive sentence types exemplified in (1)–(5) at the beginning of this chapter, as well as the unmarked non-disjunctive sentence types in (6)–(7). Then, in Section 6.5, we turn to the more ‘marked’ sentence types which were exemplified in (8)–(10).
6.4 Unmarked cases

We start with the simplest unmarked sentence type, namely a non-disjunctive declarative with falling intonation, repeated in (22):

(22) Igor speaks Spanish.

This sentence is taken to have the following logical form:

(23) \[[\text{closed decl}] [C_{\text{DECL}} \text{ Igor speaks Spanish}]\]

The translation of this logical form is \(\lnot \lnot p\), which can be simplified to just \(p\). The proposition expressed by this sentence is depicted in Figure 6.1(a). Thus, it is correctly predicted that the sentence provides the information that Igor speaks Spanish, without requesting any additional information.

Next, consider the disjunctive falling declaratives in (24) and (25):

(24) Igor speaks Spanish-or-French.
(25) Igor speaks Spanish or he speaks French.

These sentences are taken to have the following logical forms, respectively:

(26) \[[\text{closed decl}] [C_{\text{DECL}} \text{ Igor speaks Spanish or French}]\]
(27) \[[\text{closed decl}] [\lnot (C_{\text{DECL}} \text{ Igor speaks Spanish}) \lor C_{\text{DECL}} \text{ he speaks French}]\]

These logical forms have the same simplified translation, namely \(\lnot (p \lor q)\), which expresses the proposition depicted in Figure 6.1(b).
Thus, the sentences are correctly predicted to provide the information that Igor speaks Spanish or French, without requesting any additional information.

Now let us turn to interrogatives. The simplest unmarked case here is the polar question in (28).

(28) Does Igor speak Spanish↑?

This sentence is taken to have the following logical form:

(29) \[[\text{open int}] \ [C_{\text{int}} \text{ does Igor speak Spanish}]\]

The simplified translation of this logical form is \(?p\), which expresses the proposition depicted in Figure 6.1(c). Thus, the sentence is correctly predicted to request information as to whether Igor speaks Spanish, and not to provide any information.

Next, consider the disjunctive polar question in (30).

(30) Does Igor speak Spanish-or-French↑?

This sentence is taken to have the following logical form:

(31) \[[\text{open int}] \ [C_{\text{int}} \text{ does Igor speak Spanish or French}]\]

The simplified translation of this logical form is \(?!(p \lor q)\), which expresses the proposition depicted in Figure 6.1(d). Again, the sentence is predicted to be inquisitive and non-informative. In order to resolve the issue that it raises, one either needs to establish that Igor indeed speaks at least one of the two languages, or that he does not speak either.

Next, consider the open disjunctive question in (32), which involves two full clauses joined by disjunction.

(32) Does Igor speak Spanish↑ or does he speak French↑?

This sentence is taken to have the following logical form:

(33) \[[\text{open int}] \ [[C_{\text{int}} \text{ does Igor speak Spanish}] \text{ or } \ [C_{\text{int}} \text{ does he speak French}]\]]

The simplified translation of this logical form is \(?!(p \lor q)\), which expresses the proposition depicted in Figure 6.1(e). As desired, the sentence is predicted to be more inquisitive than (30): in order to resolve the issue that it raises, it is not sufficient to establish whether or not Igor speaks at least one of the two languages. Rather, it either needs to be
established that Igor speaks Spanish, or that he speaks French, or that he speaks neither.

Note in particular that (32) is not translated as \( ?p \lor ?q \), but rather as \( ?(p \lor q) \). This is a desirable result, because if it were translated as \( ?p \lor ?q \), then the account would predict that in order to resolve the issue that (32) expresses, it would be sufficient to establish that Igor does not speak Spanish, or to establish that he does not speak French. This prediction would be wrong: to resolve the issue expressed by (32), establishing that Igor does not speak either language is sufficient, but establishing that Igor does not speak Spanish (or that that he doesn’t speak French) is not. In order to achieve this result, it is crucial that the \( ? \) operator is not contributed by the interrogative clause type markers in (33). Rather, it is contributed by the incompleteness marker open, which scopes over the entire list structure.

Finally, consider the alternative question in (34), which again involves two full clauses joined by disjunction, but now with falling intonation on the second clause.

(34) Does Igor speak Spanish\(^\uparrow\) or does he speak French\(^\downarrow\)?

closed interrogative

This sentence is taken to have the following logical form:

(35) \[
\text{[\text{closed int} \quad \text{[\text{C}_{\text{int}} \quad \text{does Igor speak Spanish}] or } \quad \text{[C}_{\text{int}} \quad \text{does he speak French}]]}
\]

The translation of this logical form, on the simplified non-presuppositional account presented here, is \( p \lor q \), which expresses the proposition depicted in Figure 6.1(f). Notice that the \( ? \) operator is not invoked here because the proposition that \text{int} gets as its input is already inquisitive. Since the role of \text{int} is not to blindly apply \( ? \), but rather just to ensure inquisitiveness, it leaves the input proposition unaltered in this case. The prediction, then, is that the alternative question in (34) provides the information that Igor speaks at least one of the two languages, and raises an issue as to which of the two languages he speaks.

As anticipated, this prediction is not entirely satisfactory, because it does not capture the fact that alternative questions presuppose that exactly one of the disjuncts holds. However, as remarked at the outset, it is impossible to properly capture this fact in InqB, which concentrates exclusively on informative and inquisitive content and leaves presuppositional aspects of meaning out of consideration. Again, we refer to AnderBois (2012), Ciardelli et al. (2012) and Roelofsen (2015a) for
presuppositional extensions of \textit{InqB}, and to the latter work for a more sophisticated version of the account presented here, which does capture the presuppositions triggered by alternative questions.

Aside from this loose end, we have seen that the present account derives the basic semantic properties of all the unmarked sentence types exemplified in (1)–(7) at the beginning of the chapter. Note that the account rests on a uniform treatment of disjunction, as well as a uniform treatment of completion markers (final pitch contours), which are used both in statements and in questions. Allowing for a uniform treatment of these common building blocks is, in our view, an important virtue of inquisitive semantics.

6.5 Marked cases

We now turn to the more marked sentence types: rising declaratives (consisting of one or multiple clauses) and falling polar interrogatives. The examples we gave in (8)–(10) are repeated in (36)–(38) below.

(36) Igor speaks Spanish↑?
(37) Does Igor speak Spanish↑?
(38) Igor speaks Spanish↑ or he speaks French↑?

We will first discuss which resolution conditions these sentences are predicted to have, and then consider how their marked status may be explained.

\textit{Resolution conditions}  The rising declarative in (36) and the falling polar interrogative (37) are translated as ?p on our account, just like the corresponding rising polar interrogative in (39).

(39) Does Igor speak Spanish↑?

Thus, as desired, it is predicted that these sentences raise an issue which can be resolved by establishing either that Igor speaks Spanish, or that he does not. On the other hand, the simplified translation of (38) is ?(p ∨ q), just like the corresponding open disjunctive question in (40).

(40) Does Igor speak Spanish↑ or does he speak French↑?

Thus, it is predicted that the issue raised by (38) can be resolved in three ways: by establishing that Igor speaks Spanish, by establishing that he speaks French, or by establishing that he does not speak either of the two
languages. In this case, it is difficult to judge whether this prediction is correct, since, as discussed above, it seems quite impossible to imagine a context in which (38) could be felicitously used.

Why are these sentence types marked? Now let us consider how the marked status of these sentence types may be explained. The general idea that we will pursue, familiar from much work in neo-Gricean pragmatics and optimality theory (see, e.g., Horn, 1984; Blutner, 2000), is that an expression is perceived as marked if there is another expression that has the same semantic content and is, other things being equal, better suited to express that content. One reason for this may be that the latter expression is easier to produce; another reason may be that it has a greater chance of being interpreted as intended. This second reason will be most relevant for us.

Notice that the logical form of every sentence in (36)–(38) is either headed by [open decl] or by [closed int]. Vice versa, every sentence type whose logical form is headed by one of these two complementizer-completion-marker combinations is represented in (36)–(38), except for alternative questions, i.e., multi-clausal closed interrogatives—we will return to this momentarily. Quite generally, then, there is something marked about closed interrogatives and open declaratives. Why would this be?

In Roelofsen (2015a); Farkas and Roelofsen (2017) it is proposed that the source of this markedness lies in the fact that these sentence types are generally in competition with open interrogatives, and that the latter are generally preferred because they maximize the chance of being interpreted as intended. This is because, in many configurations, open and int have precisely the same semantic effect, and even more importantly, in these configurations the same overall interpretation would result if either open or int were to be misinterpreted as closed or decl, respectively.

Let us look at an example to make this more concrete. The open declarative in (36) and the open interrogative in (39) are both translated as ?p, and are thus predicted to have exactly the same semantic content. Now suppose that someone hears (39) in a conversation and has to determine its intended interpretation. If all goes well, the sentence is

12 Recall that we will not try here to characterize the special discourse effects and felicity conditions of rising declaratives and falling polar interrogatives; see Farkas and Roelofsen (2017) for an analysis of rising declaratives that is compatible with the account presented here.
recognized as an open interrogative—through the interrogative word order and the final rise. However, even if the sentence is mistakenly parsed as an open declarative, or as a closed interrogative, the same interpretation would still be derived. Thus, open interrogatives are very robust: if one piece breaks, the whole construction still functions as intended. This is not the case for the open declarative in (36). If this sentence is mistakenly parsed as a closed declarative, the intended interpretation would not be obtained. This explains the non-optimal, marked nature of this sentence type.

Exactly the same reasoning applies to the closed interrogative in (37). This sentence also has \( ?p \) as its translation, so it is also in competition with the open interrogative in (39). And again, it does not have the same robustness as the open interrogative, because if it is mistakenly parsed as a closed declarative, the intended interpretation is not obtained.

Finally, the markedness of the bi-clausal open declarative in (38) can be explained in a similar way as well, although here the reasoning is somewhat more involved. As noted above, (38) is predicted to be semantically equivalent with the open interrogative in (40); both are translated as \( ?(p \lor q) \). Now consider which interpretations arise if either (38) or (40) is not parsed as intended. If the open declarative in (38) is mistakenly parsed as an open interrogative it is translated as \( ?(p \lor q) \), which is still its intended interpretation, but if it is mistakenly parsed as a closed declarative it is translated as \( !(p \lor q) \), which is clearly different from \( ?(p \lor q) \).

On the other hand, if the open interrogative in (40) is mistakenly parsed as an open declarative, it is translated as \( ?(p \lor q) \), which is its intended interpretation, but if it is mistakenly parsed as a closed interrogative it is translated as \( p \lor q \), which is different from \( ?(p \lor q) \). So if we just count the number of erroneous parses that lead to misinterpretation, there is no reason to prefer the open interrogative over the open declarative in this case. If we take a closer look, however, we find that such a reason does exist.

Consider the interpretations that arise if the two sentences are misinterpreted. In the case of (40) we obtain \( p \lor q \); in the case of (38) we get \( !(p \lor q) \). Neither of these coincides with the intended interpretation, \( ?(p \lor q) \). However, it may be argued that the former misinterpretation is less consequential than the latter. To see this, note that \( p \lor q \) entails \( ?(p \lor q) \), which means that every resolution of the former is also a resolution of the latter. Thus, even if (40) is misinterpreted as \( p \lor q \), it will still be taken to request information which would, if provided
by the addressee, resolve the issue expressed by the sentence under its intended interpretation. On the other hand, !(p ∨ q) does not entail ?(p ∨ q). In fact, unlike ?(p ∨ q), !(p ∨ q) is not inquisitive at all. So if (38) is misinterpreted as !(p ∨ q), then the addressee will not be prompted to provide any information, let alone information that would resolve the issue expressed by the sentence under its intended interpretation. Thus, the potential misinterpretation of the open interrogative in (40) is less consequential than the potential misinterpretation of the open declarative in (38). This is a reason for speakers to prefer (40) over (38) when they want to express the proposition associated with ?(p ∨ q). This, then, explains the marked status of multi-clausal open declaratives like (38).

Finally, let us return to the case of alternative questions, i.e., multi-clausal closed interrogatives, which are not marked, even though uni-clausal closed interrogatives are. The reason for this is that multi-clausal closed interrogatives are not generally equivalent with the corresponding open interrogatives. So in this case there is no competition between the two types of lists.

To make this concrete again, consider the closed interrogative in (41).

(41) Does Igor speak Spanish↑ or does he speak French↓?

The simplified translation of this sentence is p ∨ q. Thus, it does not have the same semantic content as the corresponding open interrogative in (40), nor is there any other competing list type. This explains its unmarked status.

This concludes our analysis of declarative and interrogative lists in InqB. Even though there is much more to say about the linguistic properties of such lists, we hope that the bare bones account that we have presented here has succeeded in substantiating the general point that we set out to make in this chapter: a uniform treatment of connectives and intonational features across declarative and interrogative constructions is greatly facilitated by a semantic framework which treats informative and inquisitive content in an integrated way. If we want to give a uniform characterization of the role of disjunction in declarative and interrogative sentences, its lexical entry should specify its contribution to both informative and inquisitive content in full generality, independently of the kind of construction that it happens to be part of. And similarly for the relevant intonational features. Simply put, the fact that declarative and interrogative sentences are largely built up from the same parts constitutes an important piece of motivation for
inquisitive semantics, which treats informative and inquisitive content
in an integrated way, as opposed to approaches in which the standard
truth-conditional notion of meaning is maintained for declaratives
and a separate notion of meaning is invoked for interrogatives (e.g.,

6.6 Exercises

Exercise 6.1

Determine the logical form of each of the examples below, and derive, step
by step, how these logical forms are translated into InqB according to the
rules in (14) and (21). Translate the indefinite expressions a bike and a car
using existential quantifiers.

(42)  a. Martina has a bike.↓
b. Martina has a bike-or-a-car↓
c. Martina has a bike↑ or she has a car↓
d. Does Martina have a bike↑?
e. Does Martina have a bike-or-a-car↑?
f. Does Martina have a bike↑ or does she have a car↑?
g. Does Martina have a bike↑ or does she have a car↓?

Exercise 6.2

Extend the basic account given here in such a way that it predicts the
acceptability and interpretation of yes and no in response to the various
types of sentences considered.

• Data to be accounted for. Your theory should account for the acceptability
and interpretation of yes and no in response to sentences (1)–(5). In particu-
lar, it should account for the fact that:

– yes and no are both acceptable in response to (1), (4), and (5); in each case
yes means that Igor speaks Spanish or French and no means that he doesn't
speak either Spanish or French.

– yes and no are not acceptable in response to (2).

– no is acceptable in response to (3), meaning that Igor doesn't speak Spanish
or French; plain yes is not satisfactory in this case, but yes, he speaks Spanish
or yes, he speaks French are fine.

• Assumptions you can make. You can assume that:

– A sentence, besides expressing a certain proposition that captures
its informative and inquisitive content, generally also highlights a set
of propositions, which may serve as the antecedents for subsequent
anaphoric expressions.

– yes and no are such anaphoric expressions:

* Both yes and no presuppose that the previous sentence highlighted a
unique proposition.

* If this presupposition is met, yes confirms the unique highlighted propo-
sition, while no rejects it.

* If the presupposition is not met, the meaning of yes and no is not defined.

– A yes/no response is only fully satisfactory if its presupposition is met
and it resolves the issue raised by the previous sentence.

• Your task. Give a recursive definition of the set of propositions that are
highlighted by sentences in InqB. You can restrict yourself to atomic sen-
tences, $\lor$, $!$, and $?$. Then show which propositions are highlighted by (1)–
(5) according to your definition, and explain how this accounts for the
varying acceptability and interpretation of yes and no in response to these
sentences.
In the previous chapter, we have seen that the inquisitive notion of meaning allows us to obtain a uniform semantic analysis of lexical and intonational elements that occur both in declarative and in interrogative sentences. However, we assumed that the logical form of an (ordinary, falling) declarative sentence is always headed by a projection operator, !, which makes the sentence non-inquisitive. This may suggest that, as long as we are only concerned with such sentences (and, therefore, not with treating operators like disjunction in a way that works uniformly across declaratives and interrogatives), the standard truth-conditional notion of meaning serves us well enough, and keeping track of inquisitive content is an unnecessary complication.

In this chapter, we will see that this is not the case: even for sentences which are not inquisitive, and whose meaning is therefore completely determined by their truth conditions, these truth conditions may depend crucially on the inquisitive content of some constituent within the sentence. Thus, to derive the right truth conditions for the whole sentence, the inquisitive content of the sentence’s constituents must be taken into account.

We will demonstrate this based on recent experimental work by Ciardelli, Zhang, and Champollion (2017c) on counterfactual conditionals. This work shows that even if two clauses \( \varphi \) and \( \varphi' \) have exactly the same truth conditions, the counterfactuals \( \varphi > \psi \) and \( \varphi' > \psi \) may have different truth conditions. In particular, the counterfactuals (1a) and (1b) have different truth-conditions, even though their antecedents are truth-conditionally equivalent.

(1) a. If switch A or switch B was down, the light would be off.
   b. If switch A and switch B were not both up, the light would be off.

This means that it is impossible to give a compositional account of counterfactuals based on a purely truth-conditional notion of meaning.
Ciardelli et al. (2017c) show that the relevant contrast finds a natural explanation once conditionals are analysed in inquisitive semantics. Moreover, Ciardelli (2016b) argues that an inquisitive analysis of conditionals has other merits as well: on the one hand, it solves a well-known problem that classical analyses of conditionals have with disjunctive antecedents; on the other hand, it does not only allow us to interpret run-of-the-mill conditional statements, but also two other classes of conditional constructions, namely, unconditionals such as (2a, b), and conditional questions such as (3a, b).

(2)  
a. Whether they play Bach or Handel, Alice will go.  
b. Whatever they play, Alice will go.

(3)  
a. If they play Bach, will Alice go?  
b. If they played Bach, would Alice go?

In this chapter, we will present the experimental results and theoretical arguments of Ciardelli (2016b) and Ciardelli et al. (2017c) in condensed form. Section 7.1 describes the experiment and explains why the obtained results are problematic for the standard view that equates meaning with truth-conditions. Section 7.2 introduces a recipe for lifting theories of conditionals from truth-conditional semantics to inquisitive semantics, and shows how the experimental results receive a natural explanation once we combine this inquisitive lifting with suitable assumptions about the process of making counterfactual assumptions. Finally, Section 7.3 discusses various further advantages of an inquisitive treatment of conditionals.

7.1 Evidence for truth-conditional effects

7.1.1 The experiment

Imagine a long hallway with a light in the middle and with two switches, one at each end. One switch is called switch A and the other one is called switch B. As the wiring diagram in Figure 7.1 shows, the light is on whenever both switches are in the same position (both up or both down); otherwise, the light is off. Right now, switch A and switch B are both up, and the light is on. But things could be different...

Which of the following counterfactual sentences are true in this scenario?

(4)  
a. If switch A was down, the light would be off.  
b. If switch B was down, the light would be off.
c. If switch A or switch B was down, the light would be off.
d. If switch A and switch B were not both up, the light would be off.
e. If switch A and switch B were not both up, the light would be on.

Ciardelli et al. (2017c) conducted an experiment to test the intuitions of native speakers of English about this question. Participants were recruited online using Amazon’s Mechanical Turk platform; they were first shown the short text above and the diagram in Figure 7.1; they were then presented with one of the sentences in (4) and a filler sentence (one at a time, in random order), and they were asked to judge these sentences as either true, false, or indeterminate. Data from participants who failed to judge the filler correctly, or who otherwise failed to qualify for the task, were rejected. The remaining results are summarized in Table 7.1. For our purposes, the most important result is the contrast between sentences (4c) and (4d): (4c) was judged true by about 70% of participants, while only 22% of participants judged (4d) true.

7.1.2 A problem for the truth-conditional view on meaning

Assuming for the moment that the judgments found in the experiment are due to an actual difference in truth value between (4c) and (4d) in the given context, this is problematic for the standard view that equates...
meaning with truth-conditions, regardless of the particular account of conditionals one assumes. To see why, consider the clauses (5a) and (5b), corresponding to the two antecedents of (4c) and (4d).

\(5\)  
   a. Switch A or switch B is down  
   b. Switch A and switch B are not both up

Assuming that our switches can only take two positions, *up* and *down*, these clauses have the same truth conditions. If switch A or switch B is down, then clearly switch A and switch B are not both up. And conversely, if switch A and switch B are not both up, then either of them must be down. Under the view that the meaning of these clauses can be identified with their truth conditions, this means that (5a) and (5b) have the same meaning.

According to the principle of compositionality, the meaning of a sentence depends only on the meaning of its constituents and the way these constituents are combined. This implies that if, in a sentence \(\varphi\), a constituent \(c\) is replaced by another constituent \(c'\) with the same meaning, the resulting sentence \(\varphi[c'/c]\) must have the same meaning as \(\varphi\).

Now, the counterfactual (4d) can be obtained from (4c) by replacing the sentential constituent corresponding to (5a) with (5b), which has the same meaning. Therefore, (4c) and (4d) must have the same meaning, and thus the same truth conditions. It follows that, in every particular context, these counterfactuals must have the same truth value. But this is not the case: in the context described in the experiment, (4c) is true, but (4d) is not.

This shows that, in combination with the principle of compositionality, the assumption that the meaning of a sentence can be identified with its truth conditions leads to wrong empirical predictions.

### 7.1.3 Ruling out alternative explanations

Ciardelli *et al.* (2017c) strengthen the argument made in the previous section by ruling out a number of alternative explanations for the data in Table 7.1.

First, one might worry that (4c) and (4d) are judged differently in spite of actually having the same truth value, due to the interference of other factors. The main concern that motivates this worry is that participants may have judged (4d) incorrectly for one of two reasons: they may have misread the phrase ‘not both up’ as ‘both not up’, that is, as ‘both down’; or they may have been confused by the higher
processing cost of the antecedent, which involves a negation scoping over a conjunction.

Neither of these hypotheses stands up to further scrutiny. According to the first hypothesis, many participants misread 'not both up' as 'both down'. If so, we would expect many participants to judge the sentence (4e) as true, since the context explicitly specifies that the light is off when both switches are down. This is not what we observe: instead, (4e) is only judged true by about 20% of participants, just like (4d).

According to the second hypothesis, many participants fail to judge (4d) true as a consequence of some context-independent feature of this sentence, such as processing cost. This hypothesis predicts that many participants would also not judge this sentence true if the circuit had been wired differently. In a post-hoc experiment, participants were asked to judge the sentences in (4) in a modified scenario, where the light is on only when both switches are up. In this scenario, an overwhelming majority of participants (about 85%) judged (4d) to be true. The contrast between the results in the two scenarios shows that the reason why (4d) was not judged true in the main experiment does not have to do with intrinsic features of the sentence, but rather with the fact that if both switches were down, the light would not be off. For this is the only difference between the original scenario and the modified one.

Another way to resist the conclusion drawn in the previous section is to accept that the difference in truth values between (4c) and (4d) is genuine, but to deny that the antecedents of these sentences have the same truth conditions. There are two natural ways to do this: one may point out that down is not logically equivalent to not up, or hypothesize that the disjunction in the antecedent of (4c) is interpreted exclusively, i.e., as requiring that only one (and not both) of the disjuncts is true, for instance as a result of some exhaustification operation of the kind discussed by Chierchia et al. (2012).

Again, further control experiments render these explanations implausible. According to the first explanation, the contrast should vanish if the word down was replaced by the expression not up throughout the sentences in (4). A post-hoc experiment revealed that this prediction is not borne out: replacing down by not up does not modify the pattern exhibited by the results in Table 7.1.

According to the second explanation, the disjunction in the antecedent of (4c) is interpreted exclusively, possibly as a result of an exhaustification operator. If so, we would naturally expect the main disjunction in (5a) to be interpreted exclusively as well, and thus to be
judged as false or indeterminate in a scenario in which both switches are down. In a pre-test, participants were presented with a picture which displays the circuit with both switches down, and they were asked to judge the sentences (5a) and (5b) as true, false, or indeterminate. Both sentences were judged true by over 80% of participants. This shows that an exclusive reading of disjunction in (5a) is at best marginal, which makes it unlikely that it is responsible for the observed contrast.1

7.2 Conditionals in inquisitive semantics

In this section, we show that the findings discussed in the previous section have a natural explanation once we move from a truth-conditional semantic setting to inquisitive semantics. We start in Section 7.2.1 by showing how inquisitive semantics assigns the same truth-conditions but different meanings to the antecedents of (4c) and (4d), thus allowing for a compositional account that assigns different truth conditions to these counterfactuals. In Section 7.2.2, we introduce the inquisitive lifting operation developed in Ciardelli (2016b), and explain how a difference in inquisitive content between two antecedents can result in different truth conditions for the corresponding conditionals. Finally, in Section 7.2.3 we present the background theory of counterfactuals developed by Ciardelli et al. (2017c), and show that the inquisitive lifting of this theory yields the right predictions for the sentences in (4).

7.2.1 Breaking de Morgan’s law in inquisitive semantics

To see how inquisitive semantics allows us to explain the data in Table 7.1, let us first formalize our sentences in the system InqB equipped with an additional counterfactual connective $>$.2 We will assume a predicate $Ux$ for ‘$x$ is up’, an atomic sentence $O$ for ‘the light is off’, and two

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1 To maintain that exhaustive strengthening is responsible for the observed effects, one would have to assume that exhaustification takes place in conditional antecedents more often than in matrix clauses. As far as we know, there is no evidence supporting this assumption.

2 Clearly, the implication connective $\rightarrow$ provided by InqB is not suitable as a translation of counterfactuals. When applied to statements, this operator yields the same truth conditions as the standard material implication connective of classical logic. Since the antecedents of our counterfactual sentences are all false, this would immediately render all these sentences true.
7.2 CONDITIONALS IN INQUISITIVE SEMANTICS

 constants $a, b$ which refer to the two switches. We will then analyse the sentences in (4) as follows:

(4a) $\neg Ua > O$
(4b) $\neg Ub > O$
(4c) $\neg Ua \lor \neg Ub > O$
(4d) $\neg (Ua \land Ub) > O$
(4e) $\neg (Ua \land Ub) > \neg O$

Let us consider the antecedents of these counterfactuals. We will assume that our model contains four possible worlds, corresponding to the four possible configurations of the switches. The propositions expressed by the different antecedents in this model are depicted in Figure 7.2.

Crucially, in inquisitive semantics, the antecedent of (4c), $\neg Ua \lor \neg Ub$, and the antecedent of (4d), $\neg (Ua \land Ub)$, are not semantically equivalent: while the two are assigned the same truth-conditions, the former is inquisitive, while the latter is not. $\neg Ua \lor \neg Ub$ generates two alternatives, namely, the set of worlds where $A$ is down, and the set of worlds where $B$ is down. By contrast, $\neg (Ua \land Ub)$ generates a single alternative, namely, the set of worlds where the switches are not both up. This means that the problem we pointed out for truth-conditional semantics no longer arises in inquisitive semantics: the antecedents of (4c) and (4d) have different meanings, and can therefore make different semantic contributions.

7.2.2 Lifting conditionals to inquisitive semantics

We now want to explain how the difference in inquisitive content between $\neg Ua \lor \neg Ub$ and $\neg (Ua \land Ub)$ can lead to a difference in truth-conditions for the counterfactuals in which these two clauses are

---

3 Of course, we do not mean here that the logical form of a sentence like (4a) has to contain a negation operator. We could introduce another predicate $Dx$ for 'x is down', and another atomic sentence $On$ for 'the light is on'. However, we would then have to introduce some meaning postulates to enforce that $Dx$ is true exactly when $Ux$ is false, and $On$ is true exactly when $O$ is false. This would then lead to the same results that our analysis gives.
embedded as antecedents. For this, we adopt an idea of Alonso-Ovalle (2006, 2009) (see also van Rooij, 2006). We assume that an antecedent need not always specify a single counterfactual assumption; rather, when we have multiple alternatives for an antecedent, each of them is treated by the semantics as a distinct counterfactual assumption. In order for the counterfactual to be true, the consequent must follow on each of these assumptions.

To implement this idea in the inquisitive setting, Ciardelli (2016b) describes a general procedure for lifting accounts of conditionals to inquisitive semantics. This lifting procedure takes as its input a truth-conditional account of (indicative or counterfactual) conditionals, given in the form of a binary operation \( \Rightarrow \) which maps any two classical propositions \( \alpha \) and \( \gamma \) (expressed by the antecedent and the consequent of a conditional, respectively) to a third classical proposition \( \alpha \Rightarrow \gamma \). All the standard theories of counterfactual conditionals, such as selection function semantics (Stalnaker, 1968), ordering semantics (Lewis, 1973), and premise semantics (Kratzer, 1981) yield such an operation \( \Rightarrow \).

The output of the lifting procedure is an ‘inquisitivized’ version of this truth-conditional account, which interprets a conditional \( \varphi > \psi \) by means of the following support clause.\(^4\)

**Definition 7.1 (Inquisitive lifting)**

\[ s \models \varphi > \psi \quad \text{iff} \quad \forall \alpha \in \text{alt}(\varphi) \exists \gamma \in \text{alt}(\psi) \text{ such that } s \subseteq (\alpha \Rightarrow \gamma) \]

\(^4\) In each of these theories, the definition of \( \alpha \Rightarrow \gamma \) makes use of some additional piece of structure: a selection function in Stalnaker (1968), a similarity ordering in Lewis (1973), an ordering source in Kratzer (1981). However, our lifting recipe only needs access to the resulting operation on propositions—not to this underlying structure.

\(^5\) Note that the interpretation of \( \varphi > \psi \) is specified in terms of support conditions. Recall that the proposition expressed by a sentence in \( \text{InqB} \) is the set of all states that support it; see Section 4.2 for discussion. Also note that the support conditions of \( \varphi > \psi \) are formulated here in terms of \( \text{alt}(\varphi) \) and \( \text{alt}(\psi) \). This formulation assumes that \( \text{alt}(\varphi) \) and \( \text{alt}(\psi) \) completely determine the meaning of \( \varphi \) and \( \psi \), respectively, i.e., that \( |\varphi| = |s| \iff s \subseteq \alpha \text{ for some } \alpha \in \text{alt}(\varphi) \) and similarly for \( \psi \). This will indeed be the case for all the examples that we will consider in this chapter, but it does not always hold in \( \text{InqB} \) (see footnote 3 on page 20). For the general case, the support conditions of \( \varphi > \psi \) can be formulated as follows, without making reference to alternatives:

\[ s \models \varphi > \psi \quad \text{iff} \quad \forall \alpha \in [\varphi] \exists \alpha' \in [\varphi] \exists y \in [\psi] : \alpha' \supseteq \alpha \text{ and } s \subseteq (\alpha' \Rightarrow y) \]

Provided that the map \( \Rightarrow \) is upward monotonic in its second argument, which is true of all truth-conditional theories of conditionals we are aware of, this clause boils down to the clause of Definition 7.1 whenever \( \text{alt}(\varphi) \) and \( \text{alt}(\psi) \) completely determine the meaning of \( \varphi \) and \( \psi \).

\(^6\) The resulting inquisitive account is called the lifting of the original account because, while the latter operates on classical propositions, the former operates on propositions in the inquisitive semantics sense, which are objects of a higher semantic type.
When \( \varphi \) and \( \psi \) are non-inquisitive, we have \( \text{alt}(\varphi) = \{|\varphi|\} \) and \( \text{alt}(\psi) = \{|\psi|\} \), and the clause therefore boils down to:

\[
s \models \varphi > \psi \iff \forall \alpha \in \{|\varphi|\} \exists y \in \{|\psi|\} \text{ such that } s \subseteq (\alpha \Rightarrow y)
\]

Thus, the conditional \( \varphi > \psi \) is predicted to be a statement whose unique alternative is the classical proposition \( |\varphi| \Rightarrow |\psi| \) delivered by the given base account. Except for (4c), all of our counterfactuals have non-inquisitive antecedents and consequents, so they will be interpreted just as they are interpreted by the given base account. As for (4c), translated as \( \neg Ua \lor \neg Ub > O \), the clause yields the following:

\[
s \models \neg Ua \lor \neg Ub > O \iff \forall \alpha \in \{\neg Ua|, \neg Ub|\} \exists y \in \{|O|\}
\]

\[
s.t. \ s \subseteq (\alpha \Rightarrow y)
\]

\[
\iff s \subseteq |\neg Ua| \Rightarrow |O| \text{ and } s \subseteq |\neg Ub| \Rightarrow |O|
\]

\[
\iff s \subseteq (|\neg Ua| \Rightarrow |O|) \cap (|\neg Ub| \Rightarrow |O|)
\]

As in the other cases, the conditional as a whole is a statement. However, the unique alternative for it, the set \( (|\neg Ua| \Rightarrow |O|) \cap (|\neg Ua| \Rightarrow |O|) \), is not the same as the set \( |\neg Ua \lor \neg Ub| \Rightarrow |O| \) that would be delivered by applying the base account directly, without lifting it to inquisitive semantics. Rather, the lifting procedure ensures that the base account is applied twice, once for each disjunct in the antecedent, and the results are then intersected. Thus, disjunctive antecedents are interpreted as providing multiple assumptions, and \( \neg Ua \lor \neg Ub > O \) is predicted to be true just in case both \( \neg Ua > O \) and \( \neg Ub > O \) are true. This explains the strong similarity between the response pattern of (4c) and those of (4a) and (4b).

Now the majority judgments in Table 7.1 could be predicted if we could find a truth-conditional account of counterfactuals according to which (4a) and (4b) are true, but (4d) and (4e) are not. The inquisitive lift of this account would make the same predictions about these sentences, and it would also predict (4c) to be true—something that no purely truth-conditional account could do.

### 7.2.3 Background semantics for counterfactuals

Now that the problem of disentangling (4c) and (4d) is solved, one might expect that we can just take a standard account of counterfactuals, such as the ordering semantics of Lewis (1973), and lift it to inquisitive
semantics to obtain an account of our data. However, as Ciardelli et al. (2017c) discuss, this is not the case. The problem is that all standard accounts of counterfactuals validate the following entailment:

$$\neg U_a > O, \neg U_b > O \models \neg (U_a \land U_b) > O$$

Thus, regardless of how the parameters needed to interpret counterfactuals are set in these theories, they can never predict that (4a) and (4b) are true but (4d) is not. Conceptually, the problem is that all the standard theories are based on the idea that, when making a counterfactual assumption, one is required to minimize the amount of change with respect to the actual world. This means that, when counterfactually assuming that A and B are not both up, one is required to retain the fact that at least one of them is up.\(^7\) This does not seem right: when asked to consider what would happen if the switches were not both up, we are naturally lead to consider the case that just one switch was down, as well as the case that both switches were down, which explains the observed judgments for (4d) and (4e).

To solve this problem, Ciardelli et al. (2017c) adopt a different perspective: they propose to replace the minimal change requirement by a qualitative distinction between aspects of the world that are in the foreground when making a counterfactual assumption, and aspects that are in the background. The latter are held fixed in the counterfactual scenario, while the former are allowed to change, and their change is not subject to any minimality requirement. We will refer to this account of counterfactuals as background semantics.

For a simple example, consider the sentences in (6):

(6)  a. If I wore my hair longer, nobody would notice the difference.
    b. If I wore my hair much longer, people would notice the difference.

In both cases, when assuming that the antecedent is true, the length of the speaker's hair is in the foreground, and we feel no pressure to imagine it to be as close as possible to the actual length; this explains why in normal circumstances we are not inclined to judge (6a) as true. On the other hand, in both cases the fact that people are able to pick up remarkable differences in hair length is in the background, and thus it

\(^7\) This may seem like an over-simplification since, e.g., in ordering semantics, one may well stipulate that toggling two switches is not to be counted as a bigger change than toggling just one. However, this stipulation would make it impossible to account for the truth of $\neg U_a > O$ and $\neg U_b > O$. A similar argument applies to the other standard accounts.
is held fixed when making the assumption; this explains why in normal circumstances we judge (6b) as true.

Now consider again (4a), (4b), and (4d). When we make the assumption that switch A is down, the position of switch B is naturally regarded as background, and therefore held fixed. This leads us to consider a counterfactual scenario in which A is down, but B is still up. Reasoning by the laws of the circuit, we therefore conclude that the light is off, and judge (4a) as true. Of course, the prediction is analogous for (4b). Now consider the assumption that the switches were not both up: in this case, the positions of both switches are at stake, and thus foregrounded. Therefore, we have no pressure to hold either of them fixed in the counterfactual scenario. This leads us to consider counterfactual scenarios where just one switch is down as well as scenarios where both switches are down: since these two kinds of scenarios do not agree on the state of the light, neither (4d) nor (4e) are judged true.

Let us now see how an account of the kind just sketched can be formalized, and verify that the predictions we just described are indeed derived. For conciseness, we present here a simplified version of the original background semantics proposed in Ciardelli et al. (2017c). This simplified version preserves the essence of the account of our sentences, although it is limited in scope.8

The account relies on a formal notion of causal structures inspired by the literature on causal reasoning (Pearl, 2009).9 For our purposes, such structures can be defined as follows.

**Definition 7.2** (Causal structures)
A causal structure is a triple \( S = \langle V, E, L \rangle \) where:

- \( V \) is a set of atomic polar questions, the causal variables of the structure. The settings of a variable \( ?\phi \) are the sentences \( \phi \) and \( \neg \phi \).
- \( \langle V, E \rangle \) is a directed acyclic graph, whose edges encode causal influence.
- \( L \) is a set of statements, the causal laws of the structure; each causal law has the form \( \phi_1 \land \cdots \land \phi_n \rightarrow \psi \), where \( \psi \) is a setting of a variable \( v \in V \) and \( \phi_1, \ldots, \phi_n \) are settings of the parents of \( v \) in the graph \( \langle V, E \rangle \).

8 In particular, unlike the original version of the semantics, the simplified version is not equipped to deal with cases in which assumptions ‘intervene’ on causal laws, in the sense of Pearl (2009).

9 For related theories of counterfactuals using causal structures, see Schulz (2011), Kaufmann (2013), and Santorio (2016).
The electric circuit described in Section 7.1 can be modeled naturally as a causal structure as follows. The causal variables are \( ?U_a, ?U_b, \) and \( ?O \), corresponding to the states of the switches and the light. The variables \( ?U_a \) and \( ?U_b \) have causal influence on \( ?O \), but not on each other. Thus, the graph \( \langle V, E \rangle \) looks as follows:

\[ ?U_a \rightarrow ?O \leftarrow ?U_b \]

The causal laws are the following conditionals, encoding the behavior of the circuit:

\[
\begin{align*}
(7) \quad \text{a.} & \quad U_a \land U_b \rightarrow \neg O \\
\text{b.} & \quad U_a \land \neg U_b \rightarrow O \\
\text{c.} & \quad \neg U_a \land U_b \rightarrow O \\
\text{d.} & \quad \neg U_a \land \neg U_b \rightarrow \neg O
\end{align*}
\]

Within the context of a causal structure, we can associate a possible world with a set of facts—i.e., basic propositions that characterize the world. Moreover, we can equip the set of facts with some structure that reflects the causal relations between them.

**Definition 7.3 (Facts)**

A fact at a world \( w \) is a true setting of a causal variable. The set of facts at \( w \) is denoted \( F_w \). The causal graph \( \langle V, E \rangle \) naturally induces a corresponding graph on the set of facts. We say that a fact \( f \) is causally dependent on another fact \( f' \) if \( f' \) is an ancestor of \( f \) in this graph.

In our context, the facts are: that switch A is up; that switch B is up; and that the light is on. That is, \( F_w = \{ U_a, U_b, \neg O \} \). The fact that the light is on is dependent on the other facts, and no other dependencies hold.

We now want to specify, given a certain counterfactual assumption \( \alpha \), which of the facts in \( F_w \) are called into question by the assumption—and should therefore be considered as potentially different in the counterfactual scenario—and which facts can be regarded as background, and can therefore be held fixed. The basic idea is simple: an assumption \( \alpha \) calls into question those facts that are logically responsible for its falsity, as well as those facts that are causally dependent on them. The remaining facts can be regarded as background, although other factors in the context might lead to them being foregrounded as well, and thus varied in the counterfactual scenario.\(^{10}\)

\(^{10}\) Ciardelli et al. (2017c) discuss in detail the fact that seeing the filler sentence can affect the way the target sentences are judged. Among the participants who saw the target
The idea of a fact being responsible for the falsity of \( \alpha \) can be formalized as follows.

**Definition 7.4 (Contributing to the falsity of a classical proposition)**
Let \( w \in W, \alpha \subseteq W \) a classical proposition, and \( f \in F_w \) a fact in \( w \). Then, if there exists some set \( F \subseteq F_w \) such that \( F \) is consistent with \( \alpha \), but \( F \cup \{ f \} \) is inconsistent with \( \alpha \), we say that \( f \) contributes to the falsity of \( \alpha \) in \( w \).

When making a counterfactual assumption \( \alpha \), we can no longer take for granted those facts that contribute to the falsity of \( \alpha \), nor anything that is causally dependent on such facts. We say that these facts are called into question by \( \alpha \).

**Definition 7.5 (Calling a fact into question)**
A classical proposition \( \alpha \) calls into question \( f \in F_w \) if either (i) \( f \) is a fact that contributes to the falsity of \( \alpha \), or (ii) \( f \) is causally dependent on some such fact.

In our concrete setting, consider the classical proposition that switch A is down, \( |\neg Ua| \). It is easy to see that the only fact \( f \in F_w \) that contributes to the falsity of this proposition is \( Ua \). Thus, the counterfactual assumption \( |\neg Ua| \) calls into question the fact that A is up, as well as the causally dependent fact that the light is on, but it does not call into question the fact that switch B is up. Similarly, the assumption that switch B is down calls into question the fact that B is up and the fact that the light is on, but not the fact that A is up.

Now consider the classical proposition that the switches are not both up, \( |\neg (Ua \land Ub)| \). It is easy to see that the fact that A is up and the Fact sentence first, less than 20% judged (4a) and (4b) as indeterminate. By contrast, among the participants who saw the filler sentence before the target sentence, about 45% judged (4a) and (4b) as indeterminate. Background semantics offers a natural explanation for this finding. The filler sentence used in the experiment was (i), whose antecedent calls into question the positions of both switches.

(i) If switch A and switch B were both down, the light would be off.

It is natural to assume that after seeing the filler, some of the participants kept thinking of the position of switch B as foregrounded even when making the assumption that A was down. Thus, they ended up considering the possibility that both switches are down, leading to the high proportion of ‘indeterminate’ judgments for (4a) (and similarly for (4b)). For a more systematic discussion of order effects, we refer the reader to Ciardelli et al. (2017c).

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11 We say that a set of facts \( F \) is consistent with \( \alpha \) if there is some world \( w \in W \) where \( \alpha \) is true (i.e., \( w \in \alpha \)) and all facts in \( F \) are true. Notice that if some fact ‘contributes to the falsity of \( \alpha \) in \( w \)', then \( \alpha \) indeed has to be false in \( w \). Otherwise no set of facts in \( F_w \) could be inconsistent with \( \alpha \).
that B is up both contribute to the falsity of this proposition. Thus, these facts are called into question, and so is the dependent fact that the light is on. Therefore, in this case the assumption calls into question all of the facts in our scenario.

The next step is to use these notions to determine which facts can be regarded as background for a given counterfactual assumption, and thus held fixed in making the assumption and assessing its consequences. We assume that only facts that are not called into question by the assumption can be backgrounded. Furthermore, we assume a requirement to avoid gratuitous changes, and thus to avoid foregrounding anything without a reason. In the absence of contextual cues providing a reason to foreground other facts, the background will consist of all and only the facts that are not called into question. We call this the maximal background for the given assumption.\footnote{We should emphasize that the requirement to maximize the background is only a default. In interpreting a counterfactual, a hearer may foreground other facts besides those called into question by the antecedent, for instance because the possibility of those facts changing is salient in the context (for some evidence that this is indeed possible, see Section 5 of Ciardelli et al., 2017c). Here we focus on the case in which the background is maximized, since we assume that it is this default interpretation that accounts for the majority judgments about our counterfactuals.}

**Definition 7.6** (Maximal background for an assumption)

The maximal background for a classical proposition $\alpha$ at world $w$, denoted $B_w(\alpha)$, is the set of all facts which are not called into question by $\alpha$.

Any fact that is part of the background for a given counterfactual assumption is held fixed in the counterfactual scenario. In other words, in making the assumption $\alpha$, we imagine that $\alpha$ is true in addition to all the background facts.

In our scenario, we have $B_w(\neg Ua) = \{Ub\}$. This explains why, when we suppose that switch A was down in our scenario, we envisage a situation where switch A is down but switch B is up. Similarly, $B_w(\neg Ub) = \{Ua\}$: when we suppose that switch B was down, we envisage a situation where switch B is down but switch A is still up. On the other hand, since the assumption $\neg (Ua \land Ub)$ calls all facts in $F_w$ into question, we have $B_w(\neg (Ua \land Ub)) = \emptyset$, which means that when we suppose that the switches were not both up, no fact carries over from the actual state of affairs to the counterfactual scenario.

We can now define the information state that results from making a counterfactual assumption $\alpha$ in a certain world $w$. This is the
information determined by the assumption itself, the background facts, and the underlying causal laws. Under a maximal background interpretation, this amounts to the following.

**Definition 7.7** (Information state resulting from an assumption)
The information state that results from making an assumption \( \alpha \) at world \( w \), denoted \( S_w(\alpha) \), is the set of worlds in which the following are true: (i) the classical proposition \( \alpha \); (ii) all facts in \( B_w(\alpha) \); and (iii) all laws in \( L \).

The last step is to specify at which worlds the classical proposition \( \alpha \rightarrow \gamma \) is true: this holds if the state that results from making the assumption, \( S_w(\alpha) \), supports the conclusion \( \gamma \), that is, if \( S_w(\alpha) \subseteq \gamma \).

**Definition 7.8** (Truth-conditional recipe for counterfactuals)
Given two classical propositions \( \alpha \) and \( \gamma \), the counterfactual proposition \( \alpha \rightarrow \gamma \) is true at a world \( w \) in case \( S_w(\alpha) \subseteq \gamma \).

This completes the description of the truth-conditional map \( \rightarrow \) that we are going to use as the basis for our inquisitive account. Let us now check that the inquisitive account that results from lifting this map correctly predicts which of the counterfactuals in (4) are true in our scenario.

First consider the counterfactual assumption that switch A was down, \( \neg U_a \). We saw that \( B_w(\neg U_a) = \{ U_b \} \). Take any world \( v \in S_w(\neg U_a) \): at world \( v \), (i) our assumption \( \neg U_a \) is true, that is, switch A is down; (ii) the background facts are true, that is, switch B is up; (iii) all causal laws are true, in particular the law \( \neg U_a \land U_b \rightarrow O \). Clearly, \( O \) must then be true in \( v \). This shows that \( S_w(\neg U_a) \subseteq O \), which means that \( \neg U_a \implies O \) is true at \( w \). Since \( \neg U_a \implies O \) is the unique alternative that our inquisitive account assigns to \( \neg U_a > O \), this counterfactual is correctly predicted to be true. Of course, the truth of the counterfactual \( \neg U_b > O \) is predicted in an analogous way.

Now consider the counterfactual \( \neg U_a \lor \neg U_b > O \). We saw in Section 7.2.2 that our inquisitive lifting account assigns a unique alternative to this sentence, namely, the intersection \( (\neg U_a \implies O) \land (\neg U_b \implies O) \). Since we have just seen that \( w \) belongs to both sets \( \neg U_a \implies O \) and \( \neg U_b \implies O \), \( w \) also belongs to their intersection. Thus, \( \neg U_a \lor \neg U_b > O \) is predicted to be true.

Finally, consider the counterfactuals \( \neg (U_a \land U_b) > O \) and \( \neg (U_a \land U_b) > \neg O \). We saw that \( B_w(\neg (U_a \land U_b)) = \emptyset \). Now, consider the state \( S_w(\neg (U_a \land U_b)) \): this state consists of those worlds where the switches are not both up, and the causal laws hold; thus, this state contains
worlds where only one switch is down and the light is off, as well as worlds where both switches are down and the light is on. Therefore, $S_w(|\neg(Ua \land Ub)|) \not\subseteq |O|$ and $S_w(|\neg((Ua \land Ub))|) \not\subseteq |\neg O|$, which means that neither $|(Ua \land Ub)| \Rightarrow |O|$ nor $|(Ua \land Ub)| \Rightarrow |\neg O|$ are true at $w$. Since these are, respectively, the unique alternative for $\neg(Ua \land Ub) > O$ and the unique alternative for $\neg(Ua \land Ub) > \neg O$, we predict that neither of these counterfactuals is true in our scenario.

Summing up, combining the background semantics for counterfactuals described in this section with the inquisitive lifting procedure described in Section 7.2.2 we obtain an account that accurately predicts which of the counterfactuals in (4) are true in the given scenario. The crucial ingredients of this account are (i) the fine-grained notion of meaning given by inquisitive semantics, (ii) an account of conditionals which is sensitive to inquisitive content, and (iii) a procedure for making counterfactual assumptions which is not constrained by the requirement to minimize the difference with respect to the actual world.13

7.3 Further benefits

In the previous section, we have seen how any truth-conditional account of conditionals, whether indicative or counterfactual, can be lifted to inquisitive semantics. Moreover, we have applied this lifting procedure to a particular truth-conditional account of counterfactuals in order to explain the experimental findings in Table 7.1. In this section, based on Ciardelli (2016b), we will demonstrate some further general benefits of the lifting procedure. We will see that no matter what truth-conditional account of (indicative or counterfactual) conditionals we take as our starting point, the lifted inquisitive account will improve on it in three ways: first, it will give a more satisfactory account of conditionals with disjunctive antecedents, avoiding a shortcoming which affects all truth-conditional accounts; second, it will allow us to interpret not only standard if-then conditionals, but also so-called unconditionals; and

13 The semantics described here is only concerned with predicting when a sentence is true. For a complete account of the data in Table 7.1, one would have to complement it with a component that explains when non-true sentences are judged as indeterminate, as opposed to simply false. It is natural to suppose that ‘indeterminate’ judgments result from the failure of a homogeneity presupposition to the effect that making a counterfactual assumption should lead to a state which settles whether the consequent is true (von Fintel, 1997). However, the issue of how failures of semantic presuppositions are reflected in truth value intuitions is a notoriously tricky one (see von Fintel, 2004).
finally, it will allow us to interpret not only conditional statements, but also conditional questions. We will consider each of these topics in a separate sub-section.

7.3.1 Simplification of disjunctive antecedents

Consider the sentences in (8). One seems justified in inferring (8b) from (8a), but certainly not in inferring (8c) from (8b).

(8) a. If Alice or Bea invited Charlie, he would go.
   b. If Alice invited Charlie, he would go.
   c. If Alice invited Charlie and then canceled, he would go.

The inference from (8a) to (8b) is an instance of a principle called simplification of disjunctive antecedents (SDA); the inference from (8b) to (8c) is an instance of a principle called strengthening of the antecedent (SA).

\[
\frac{A \lor B > C}{A > C} \quad (\text{SDA}) \quad \frac{A > C}{A \land B > C} \quad (\text{SA})
\]

Intuitively, SDA is valid. Indeed, a conditional like (8a) seems to mean exactly the same as the conjunction in (9).

(9) If Alice invites Charlie, he will go, and if Bea invites him, he will go.

However, classical theories of counterfactuals, such as Stalnaker (1968); Lewis (1973) and Kratzer (1981), fail to validate this principle. This has been widely regarded as a problem for these theories (see, e.g., Fine, 1975; Nute, 1975; Ellis et al., 1977; Alonso-Ovalle, 2009; Fine, 2012) and it has also been clear since Fine (1975) that this problem is more than an accidental shortcoming. Indeed, based on a truth-conditional view on meaning and the classical treatment of connectives, a compositional account that validates SDA is bound to validate SA as well, and this is undesirable in view of the invalid inference from (8b) to (8c).\footnote{In fact, the problem is not limited to counterfactuals, but concerns conditionals more generally. The intuitions about the indicative conditionals in (i) are exactly the same as those for the corresponding counterfactuals in (8).

(i) a. If Alice or Bea invites Charlie, he will go.
    b. If Alice invites Charlie, he will go.
    c. If Alice invites Charlie and then cancels, he will go.

The proof given by Fine (1975) for counterfactuals also shows that a compositional account of indicative conditionals based on truth-conditions is bound to make SDA and SA inter-derivable.}

In recent years, this problem has motivated approaches to counterfactuals which rely on a more fine-grained semantic representation of
antecedents than the truth-conditional one. Perhaps the most prominent account of this kind is due to Alonso-Ovalle (2006, 2009), which we already mentioned above as an inspiration for the inquisitive lifting procedure (for accounts in the same spirit, see also van Rooij, 2006; Fine, 2012; Willer, 2015). The fundamental idea of this account is that disjunctive sentences denote sets of classical propositions, rather than single propositions, and that each proposition in the set serves as a separate counterfactual assumption. Clearly, this approach validates SDA: evaluating a counterfactual with a disjunctive antecedent, \( A \lor B > C \), effectively amounts to evaluating the conjunction \( (A > C) \land (B > C) \).

On the other hand, SA is invalid: for non-disjunctive antecedents, Alonso-Ovalle's account coincides with the ordering semantics of Lewis (1973), which invalidates SA.

The inquisitive lifting recipe achieves essentially the same: when an antecedent is associated with multiple alternatives, the lifted account leads us to run the base account separately for each of these alternatives. This holds in particular for disjunctive antecedents, which typically present one alternative for each disjunct. Thus, no matter what account of conditionals we take as our starting point, the lifting of this account will interpret \( A \lor B > C \) as equivalent with \( (A > C) \land (B > C) \), validating SDA. On the other hand, if the base account does not validate SA, neither will its lifting, since the two will coincide in the absence of inquisitiveness. Thus, inquisitive semantics provides a way to disentangle SDA from SA and to avoid one of the central problems faced by standard theories of conditionals.

\[ 15 \text{ Disjunctive antecedents where one of the disjuncts entails the other, either logically or contextually, form an exception to this claim. If } A, B \text{ are atomic sentences with } |A| \subseteq |B|, \text{ then } A \lor B \equiv B \text{ in inquisitive semantics, and as a consequence, } (A \lor B > C) \equiv (B > C). \text{ We take this to be a welcome result. For consider a conditional of this special form, such as (i): } \]

(i) If we hire an American or a Californian, we should arrange a visa.

This sentence is odd if uttered by someone who is aware of the fact that Californians are Americans. This is commonly explained in terms of a ban against logical forms that contain structural redundancy (Katzir and Singh, 2013; Meyer, 2014). In Alonso-Ovalle's account, this explanation is no longer available: even when \( |A| \subseteq |B| \), we have \( (A \lor B > C) \equiv (B > C) \), so a sentence like (i) does not involve any structural redundancy. By contrast, on our account we have that \( (A \lor B > C) \equiv (B > C) \), which allows us to preserve the standard explanation for the oddity of (i). This observation is not specific to conditionals, but it points to an underlying difference between inquisitive semantics and alternative semantics, the framework in which Alonso-Ovalle's account is cast. For extensive discussion of this point, see Ciardelli and Roelofsen (2017a). We will also come back to this in Section 9.1, where we compare inquisitive and alternative semantics in detail.
With respect to Alonso-Ovalle’s own account, the inquisitive treatment of conditionals can be seen as a generalization in three different ways. First, on the inquisitive approach, disjunction is not treated as a special, non-standard connective; instead, all connectives are taken to operate on inquisitive propositions, rather than on classical propositions. As we saw in Chapter 4, this allows us to retain a principled and well-behaved theory of propositional connectives, which preserves the attractive features of the classical theory.

Second, Alonso-Ovalle’s account is based on a specific account of conditionals, namely, the ordering semantics of Lewis (1973). By contrast, the inquisitive lifting recipe can be applied to any base account of conditionals, provided that it is compositional and operates in a truth-conditional setting. In the previous section, we have already made use of this degree of freedom. As we saw, independently of the issue of disjunctive antecedents, minimal change theories could not possibly predict the majority judgments in Table 7.1, as these run against the very logic of these theories. The modularity of the inquisitive lifting strategy allowed us to disentangle the problem of dealing with disjunctive antecedents from the problem of determining the right procedure for making counterfactual assumptions.

Finally, inquisitive lifting is not specifically designed to deal with disjunctive antecedents; rather, it provides a general treatment of the interaction between conditionals and inquisitiveness—an interaction which is manifested not just in conditionals with disjunctive antecedents, but in other classes of conditional sentences as well, as we will discuss in Section 7.3.2 and 7.3.3.

Before turning to the next topic, let us spend a few words on some examples that seem to show that SDA is not in fact generally valid. These examples have a special form, with the consequent coinciding with one of the disjuncts in the antecedent. The most famous such example is (10), due to McKay and Van Inwagen (1977):

(10) If Spain had fought with the Axis or the Allies in WWII, she would have fought with the Axis.

This sentence seems true, even though it is certainly not the case that if Spain had fought with the Allies she would have fought with the Axis. On a standard theory like the one of Lewis (1973), the truth of this sentence would be explained by saying that some worlds where Spain fought with the Axis are more similar to the actual world than any world
where Spain fought with the Allies. However, this diagnosis leads us to expect that (11a) is true, since it effectively boils down to (11b).

(11) a. If Spain had fought with the Axis or the Allies in WWII, Germany would have been pleased.
    b. If Spain had fought with the Axis, Germany would have been pleased.

As Nute (1980) notes, this is wrong: (11a) is naturally interpreted as implying that Germany would have been pleased if Spain fought with the Allies, in accordance with SDA. This problem, together with the fact that these counterexamples have a very special form, suggests that these cases involve some kind of anomaly.

In our inquisitive account, these counterexamples could be accounted for by stipulating that it is in principle possible to insert a projection operator ! in the antecedent. Thus, (10) would be translated as !\((Ax \lor Al) > Ax\) and analysed as a basic conditional with a non-inquisitive antecedent, which would block SDA in this case. However, the possibility to insert ! should be restricted, in order to account for the apparent lack of ambiguity of ordinary conditionals such as (8a) and (11a). One way of explaining why ! is inserted in (10) is based on the observation that a logical form such as \(Ax \lor Al > Ax\) is equivalent with the simpler form \(Al > Ax\). Assuming a general ban against structural redundancy, of the kind proposed by Meyer (2014), this would make the logical form \(Ax \lor Al > Ax\) unavailable for a conditional such as (10), justifying the insertion of ! as a repair strategy. This explanation would account for why SDA only seems to fail in sentences where the consequent coincides with one of the disjuncts in the antecedent.

7.3.2 Unconditionals

In the previous section, we mentioned that our account derives the behavior of disjunctive antecedents as a particular case of a more general pattern of interaction between conditionals and inquisitiveness. Another class of sentences in which this interaction is manifested is that of unconditionals. These are sentences such as the following:

(12) a. Whether they play Bach or not, Alice will go.
    b. Whether they play Bach or Handel, Alice will go.
    c. Whatever they play, Alice will go.

\[16\text{ We are assuming here that the underlying account of conditionals makes any proposition } \alpha \Rightarrow \alpha \text{ tautological. This is a minimal desideratum for an account of conditionals, both indicative and counterfactual, and it holds in any theory that we are aware of.}\]
Following Rawlins (2008), we will analyse unconditionals as conditional constructions where the ‘antecedent’ is an interrogative clause. According to the compositional account given in Chapter 6, we translate the polar interrogative whether they play Bach or not as \(Pb \lor \neg Pb\), which is equivalent to \(?Pb\), and the disjunctive interrogative whether they play Bach or Handel as \(Pb \lor Ph\). Moreover, we assume that the antecedent of (12c) corresponds to the interrogative what they play, which we translate as \(\exists x Px\). This gives the following translation for our unconditionals.

\[
(\text{13}) \quad \begin{align*}
\text{a. Whether they play Bach or not, Alice will go.} & \quad ?Pb > G \\
\text{b. Whether they play Bach or Handel, Alice will go.} & \quad Pb \lor Ph > G \\
\text{c. Whatever they play, Alice will go.} & \quad \exists x Px > G
\end{align*}
\]

Let us now consider the semantics that our inquisitive account of conditionals assigns to these sentences. Let us start with (12a). The antecedent is inquisitive, while the consequent is not: \(\text{alt}(?Pb) = \{|Pb|, |\neg Pb|\}\), \(\text{alt}(G) = \{|G|\}\). Our support clause gives:

\[
s \models ?Pb > G \iff \forall \alpha \in \{|Pb|, |\neg Pb|\} \exists y \in \{|G|\} \text{ such that } s \subseteq (\alpha \Rightarrow y) \\
\iff s \subseteq (|Pb| \Rightarrow |G|) \cap (|\neg Pb| \Rightarrow |G|) \\
\iff s \models (Pb > G) \land (\neg Pb > G)
\]

According to this analysis, (12a) is non-inquisitive, and it is true in case Alice will go if they play Bach, and she will go if they do not. This is precisely the analysis we expect for the unconditional (12a). Similarly, for (12b) and (12c) we obtain the following predictions:

\[
s \models Pb \lor Ph > G \iff \forall \alpha \in \{|Pb|, |Ph|\} \exists y \in \{|G|\} \text{ such that } s \subseteq (\alpha \Rightarrow y) \\
\iff s \subseteq (|Pb| \Rightarrow |G|) \cap (|Ph| \Rightarrow |G|) \\
\iff s \models (Pb > G) \land (Ph > G)
\]

\[
s \models \exists x Px > G \iff \forall \alpha \in \{|Pd| \mid d \in D\} \exists y \in \{|G|\} \text{ such that } s \subseteq (\alpha \Rightarrow y) \\
\iff s \subseteq \bigcap_{d \in D} (|Pd| \Rightarrow |G|) \\
\iff s \models \forall x (Px > G)
\]

It is derived that both (12b) and (12c) are non-inquisitive; further, (12b) is true in case Alice will go both if they play Bach and if they play

\[17\] Just as in Chapter 6, we disregard at this point the presuppositional component of these sentences. We will return to this later on in this section.
Handel, and (12c) is true if for every x in the domain, Alice will go if they play x. Again, these are indeed the natural truth conditions for these sentences.

Thus, our inquisitive account of conditionals extends naturally to a general analysis of unconditional sentences. The resulting analysis is in line with the one proposed by Rawlins (2008), and shares its core idea. However, a nice feature of the inquisitive approach is that it is modular: it does not commit us to a specific account of conditionals, but is compatible with a wide range of accounts. A second advantage is that nothing special had to be stipulated to analyse unconditionals: the desired analysis follows for free from the semantics of questions and the conditional operator, once unconditionals are analysed as conditionals. Thus, the approach is not merely descriptive, but also has some explanatory power. Another way to put this last point is this: we have given a uniform account of disjunctive and interrogative antecedents as introducing multiple assumptions, and provided an explanation for this commonality based on a feature shared by disjunctive and interrogative clauses, namely, inquisitiveness.

One may complain that, in giving this uniform explanation, we have gone too far: at this point, we have given exactly the same translation for the standard if-conditional in (17a) and the unconditional in (17b).

(17) a. If they play Bach or Handel, Alice will go. \[ Pb \lor Ph > G \]
    b. Whether they play Bach or Handel, Alice will go. \[ Pb \lor Ph > G \]

There is a sense in which this prediction is correct. Both (17a) and (17b) are not inquisitive, and they have the same truth conditions: both are true in case Alice will go if they play Bach, and also if they play Handel. Yet, intuitively there is also a difference between these sentences, which is not reflected in our translation.

The idea pursued in Ciardelli (2016b), proposed already by Zaefferer (1991), is that the difference between (17a) and (17b) is one of presupposition: the unconditional in (17b) presupposes that they will play either Bach or Handel, whereas the conditional in (17a) lacks this presupposition. This can be seen with the following pair of examples, the first of which is adapted from Zaefferer:

(15) a. The meeting might be in London; but if it is in Rome or in Paris, Alice will be there.
    b. ??The meeting might be in London; but whether it is in Rome or in Paris, Alice will be there.
(16)  a. Whether the baby is a boy or a girl, they will be a happy family.
    b. ??If the baby is a boy or a girl, they will be a happy family.

In the case of (15), the first sentence in the discourse indicates that the speaker cannot presuppose that the meeting is in Rome or in Paris. It is then odd for her to continue with an unconditional which carries this presupposition. By contrast, in the case of (16), it would be natural for a speaker to presuppose that the baby will be a boy or a girl. This makes her use of a standard conditional form odd as a result of the principle maximize presupposition, which requires speakers to prefer equivalent forms with stronger presuppositions whenever these presuppositions are satisfied (see Ciardelli, 2016b, for a more detailed discussion of these data).\footnote{Ciardelli (2016b) also notes that the maximize presupposition principle explains the oddness of a conditional like If they play Bach or they don’t, Alice will go. Since one can always presuppose that either they will or they will not play Bach, the principle requires that, in any context, a speaker should choose the corresponding unconditional form, Whether they play Bach or not, Alice will go.}

Importantly, in the analysis we described, this semantic difference between standard conditionals and unconditionals does not have to be stipulated, but can be derived from two standard generalizations about the presuppositions of interrogatives, and the way presuppositions project from conditional antecedents.

1. Interrogative clauses presuppose that one of their alternatives is true.
   (see, e.g., Belnap, 1966)\footnote{For further discussion of this generalization, see Ciardelli (2016d, pp. 21–7).}
2. Conditionals inherit the presuppositions of their antecedent.
   (see, e.g., Karttunen, 1973, 1974)

Since we view unconditionals as conditionals with an interrogative clause as their antecedent, it follows from (1) and (2) that unconditionals always presuppose that one of the alternatives for their antecedent is true. For a formalization of these ideas in a system that captures presuppositions, see Ciardelli (2016b).

### 7.3.3 Conditional questions

Another class of sentences which involve the interplay of conditionals and inquisitiveness is given by conditional questions, such as those in (17) and (18).

(17)  a. If Alice goes to the concert, will they play Bach?
    b. If Alice goes to the concert, what will they play?
(18) a. If Alice went to the concert, would they play Bach?
b. If Alice went to the concert, what would they play?

Standard theories of conditionals, being couched in a truth-conditional semantic framework, cannot be directly applied to analyse these sentences. By contrast, the inquisitive lifting of such theories can be applied directly to these questions, yielding natural results. Let us see how. Here, we will not take a stance on what the semantic difference is between indicative and counterfactual conditionals; we will just suppose that we are given two maps \( \Rightarrow_i \) and \( \Rightarrow_c \) which correspond to these two different classes of conditionals, and we will assume two operators \( >_i \) and \( >_c \) which are interpreted by lifting these maps to inquisitive semantics.\(^{20}\)

As above, we translate the clause whether they play Bach as \( ?Pb \), and the clause what they play as \( \exists xPx \). This gives the following translations for our sentences:

\[
\begin{align*}
(17a) & \quad G >_i ?Pb & (18a) & \quad G >_c ?Pb \\
(17b) & \quad G >_i \exists xPx & (18b) & \quad G >_c \exists xPx
\end{align*}
\]

Let us now see what predictions this yields for the conditional questions in (17) and (18). Since the lifting recipe works in the same way for indicative and counterfactual conditionals, we will suppress subscripts in the derivation. Let us start with the conditional polar questions in (17a) and (18a).

\[
s \models G > ?Pb \iff \forall \alpha \in [\vert G \vert] \exists y \in [\vert Pb \vert, \vert \neg Pb \vert] \text{ such that } s \subseteq (\alpha \Rightarrow y) \\
\iff s \subseteq [\vert G \vert] \Rightarrow [\vert Pb \vert] \text{ or } s \subseteq [\vert G \vert] \Rightarrow [\neg Pb] \\
\iff s \models G > Pb \text{ or } s \models G > \neg Pb
\]

Thus, (17a) and (18a) are predicted to be inquisitive. A state supports \( G > ?Pb \) iff it supports \( G >_i Pb \), or it supports \( G >_i \neg Pb \); this means that in order to resolve (17a), one must establish either that if Alice goes they will play Bach, or that if Alice goes they will not play Bach. These are precisely the resolution conditions that we expect for (17a). Similarly, (18a) is supported iff either of \( G >_c Pb \) and \( G >_c \neg Pb \) is supported, which again gives the natural resolution conditions for this question.

Now let us consider the conditional wh-questions in (17b) and (18b).

\(^{20}\) This assumption does not preclude the possibility of having a uniform semantics for both classes of conditionals: in this case, the maps \( \Rightarrow_i \) and \( \Rightarrow_c \) will be derived from the same underlying account, perhaps by setting some parameters differently in the two cases.
Thus, (17b) and (18b) are predicted to be inquisitive. An information state supports $G \supset P$ if it supports $G \supset P$ for some $d \in D$; this means that in order to resolve (17b), one must establish for some specific $d$ that if Alice goes, they will play $d$. Similarly, (18b) is supported iff $G \supset P$ is supported for some $d \in D$. Again, these are precisely the resolution conditions that we would intuitively assign to these questions.

These examples illustrate how lifting an account of conditionals to inquisitive semantics immediately yields an extension of this account to conditional questions. This approach differs from previous accounts of conditional questions such as Velissaratou (2000) and Isaacs and Rawlins (2008), which focus on indicative conditional questions like those in (17) and cannot be used directly to analyse counterfactual conditional questions like those in (18). As we have seen, inquisitive lifting applies uniformly to indicative and counterfactual questions. Additionally, inquisitive lifting leaves us with a choice as to the underlying theory of conditionals that we use to interpret these questions.

Before concluding this section, an important issue remains to be addressed. At this point, the reader might be worried that the conditional statement (19a) might end up being assigned the same meaning as the conditional question (19b).

\begin{enumerate}
\item a. If Alice goes, they will play Bach or Handel.
\item b. If Alice goes, will they play Bach, or Handel?
\end{enumerate}

This problem does not arise, however, since as discussed in Chapter 6, the LF of a declarative or interrogative clause always involves a complementizer which contributes a corresponding operator $\lnot$ or $?$. Thus, we translate (19a) to our formal language as $G \supset \lnot(Pb \lor Ph)$, and we translate (19b) as $G \supset (?)(Pb \lor Ph)$, which is equivalent to $G \supset (Pb \lor Ph)$. In all the other examples discussed in this chapter inserting the operators $\lnot$ and (?) in the main clause would have a vacuous effect, which is why we could safely disregard these operators. However, it is crucial for us to assume that the interpretation of an ‘if’ clause does not involve any projection operator. This is justified by the observation that if-clauses are syntactically distinct from both declarative and interrogative clauses; the former are headed by the complementizer ‘if’, while the latter are
taken to be headed by declarative or interrogative complementizers, which in English main clauses are not lexicalized, but affect word order.  

Summing up, in this section we saw that lifting an account of conditionals to inquisitive semantics leads to an account which improves on the original one in various ways: first, it gives a more satisfactory treatment of disjunctive antecedents, which are interpreted as providing multiple assumptions; second, it extends the scope of the original account beyond standard conditional statements, allowing us to analyse two other classes of conditional constructions: unconditionals, and conditional questions.

7.4 Summary

Our main goal in this chapter was to show that inquisitive content is relevant even for phenomena that have no obvious link to questions, and that the inquisitive content of a constituent can sometimes play a crucial role in determining the truth conditions of a sentence. We have illustrated this point with conditionals, which provide an especially interesting and rich domain of application. In this domain, taking inquisitive content into account provides a natural explanation for some otherwise puzzling data (such as those in Table 7.1), solves some long-standing logical problems (the inter-derivability between SDA and SA), and allows for a substantial extension of the scope of standard theories (bringing unconditionals and conditional questions within reach).  

We think that conditionals are not an isolated case, but only one of many environments where inquisitive content plays a role. To give one example, it has been argued by Simons (2005), Aloni (2007), and Willer (2017), among others, that something like inquisitive content is responsible for the free-choice inferences triggered by disjunctions under

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21 In the case of unconditionals, we assume that the antecedent is an interrogative, and thus involves the operator ⟨⟩. In all the cases that we discussed in this chapter, the presence of this operator would not affect the meaning we predict. Nevertheless, as we discussed in Section 7.3.2, we take the fact that the antecedent is interrogative to be responsible for the existential presupposition associated with unconditionals. In a presuppositional refinement of our analysis of interrogative complementizers (see Roelofsen, 2015a), this presupposition would be derived automatically.

22 The analysis of indicative conditional statements and questions has also given rise to further refinements of the basic inquisitive notion of meaning presented here (Groenendijk and Roelofsen, 2010, 2015; Aher and Groenendijk, 2015). These refinements address empirical issues that are orthogonal to the ones considered here.
modals, exemplified in (20), which are not predicted under standard theories of modals.

(20)  
\( a.\) Alice might speak Dutch or French.  \( \Diamond(p \lor q) \)  
\( b. So, Alice might speak Dutch. \)  \( \sim \Diamond p \)

Interestingly, the same contrast between disjunctions and negated conjunctions that we discussed above in the case of counterfactual antecedents is found in the domain of modals. For instance, (21a) does not have the free choice inference in (21b). To see this, consider a context where we are looking for someone to translate from Dutch to French, and where it is known that Alice speaks Dutch, but it is not known whether she also speaks French; in this context, (21a) is true, but (21b) is not.

(21)  
\( a.\) Alice might not speak both Dutch and French.  \( \Diamond\neg(p \land q) \)  
\( b. \) #Alice might not speak Dutch.  \( \sim \Diamond \neg p \)

If free choice inferences stem from the presence of multiple alternatives, then the contrast is expected from the inquisitive semantics perspective, since disjunctions are typically inquisitive, but negated conjunctions are not.

Clearly, more work, both empirical and theoretical, is needed to investigate exactly in which linguistic environments inquisitive content plays a role, and to provide formal accounts of the relevant phenomena.

### 7.5 Exercises

**Exercise 7.1 Lifting material implication**

Show that inquisitive implication is the lifting of material implication. That is, show that if \( \Rightarrow \) is defined as material implication (i.e., for every two classical propositions \( p \) and \( q \), \( p \Rightarrow q \) amounts to \( \overline{p} \cup q \)), then the support conditions assigned to \( \phi > \psi \) by the inquisitive lifting recipe coincide with the support conditions of \( \phi \rightarrow \psi \) in InqB.

**Exercise 7.2 Background semantics**

Consider sentence (22) in the following two scenarios (Tichý, 1976):

- **Context 1:** Jones has the following habits as regards wearing his hat. Bad weather invariably induces him to wear his hat. Fine weather, on the other hand, affects him neither way: on fine days he puts his hat on or leaves it on the peg, completely at random. Suppose moreover that actually the weather is bad, so Jones is wearing his hat.
• **Context 2:** Jones always flips a coin before he opens the curtains to see what the weather is like. Heads means he is going to wear his hat in case the weather is fine, whereas tails means he is not going to wear his hat in that case. Like above, bad weather invariably makes him wear his hat. Today heads came up when he flipped the coin, and it is raining. So Jones is wearing his hat.

(22) If the weather was fine, Jones would be wearing his hat.

Intuitively, the sentence is true in context 2 but not in context 1. Show how this is derived in background semantics of counterfactuals, modeling the causal structure of each context.

**Exercise 7.3 Quantification in the antecedent of a counterfactual**

Consider an electrical circuit with four switches and one light. The light is on if and only if an even number of switches is up. Currently, all switches are up, so the light is on. Now consider the following sentences:

(23) If any of the switches was down, the light would be off.
(24) If the switches were not all up, the light would be off.

Intuitively, (23) is true in the given scenario, but (24) is not. Suppose that the sentences are translated as \( (∃x.¬Ux) > O \) and \( (∼∀x.Ux) > O \), respectively.

1. Show that the given intuitions cannot be captured by any truth-conditional compositional account of counterfactuals.

2. Show that they are captured by the inquisitive account described above.

**Exercise 7.4 Conditional questions with disjunctive antecedents**

Consider the following indicative conditional question:

(25) If Alice goes to London or to Paris, will she take the train?

1. Translate the sentence into a suitable first-order logical language.

2. Assuming a truth-conditional map \( ⇒ \) for indicative conditionals, derive the support conditions that the inquisitive lifting of \( ⇒ \) assigns to the question in (25).

3. What does this predict about the circumstances under which the question is resolved?
Propositional attitudes

In the previous chapters we have seen that inquisitive semantics provides a new notion of semantic content, which does not just embody informative content but also inquisitive content, as well as a new notion of conversational contexts, which does not only capture the information that has been established in the conversation so far but also the issues that have been raised. In this chapter we will show that the framework also gives rise to a new view on propositional attitudes, especially those that are relevant for information exchange. Namely, besides the familiar information-directed attitudes like knowing and believing it also allows us to model issue-directed attitudes like wondering and being curious.

A perspicuous and widely adopted formal treatment of information-directed attitudes is provided by epistemic logic (EL), sometimes also called the logic of knowledge and belief, which has its roots in the work of Hintikka (1962) and has been further developed by many authors in subsequent work (see, e.g., Fagin et al., 1995; van Ditmarsch et al., 2007; van Benthem, 2011). In this framework, the information state of an agent is modeled as a set of possible worlds, namely those worlds that are compatible with the information available to the agent. As we have seen, this notion of information states also plays an important role in inquisitive semantics. However, while the information-directed attitudes of an agent can be captured in terms of her information state, this clearly does not hold for issue-directed attitudes. In order to capture the attitude of wondering, we need a description of the agent's inquisitive state, i.e., a representation of the issues that she entertains.

To this end, we will define an inquisitive epistemic logic (IEL, Ciardelli and Roelofsen (2015)), which brings together ideas from standard EL and InqB. This logic enriches InqB with two modal operators: $K$, which is used to talk about the agents' knowledge, and $E$, which is used to talk about the issues that the agent entertains.

One purpose of IEL is to serve as a formal framework to describe and reason about information- and issue-directed attitudes as such.
However, this is not the only purpose. Of equal importance, it also provides a basic semantic treatment of verbs in natural languages that are used to report such attitudes. In English, such verbs include *know* and *wonder*, and many other languages have verbs that fulfil precisely the same purpose. While the semantics of *know* and its cross-linguistic kin has been considered extensively, its treatment in IEL differs from most previous accounts in that it deals completely uniformly with cases where *know* takes a declarative complement, as in (1a), and cases where it takes an interrogative complement, as in (1b–d).

(1) a. Alice knows that Bob is coming.
   b. Alice knows whether Bob is coming.
   c. Alice knows whether Bob or Charlie is coming.
   d. Alice knows who is coming.

As for *wonder*, IEL does not only capture its interpretation when taking an interrogative complement, as in (2a) below, but it also provides an explanation of the fact that the verb cannot take a declarative complement, illustrated in (2b).

(2) a. Alice wonders whether Bob is coming.
   b. *Alice wonders that Bob is coming.1

We will proceed as follows. Section 8.1 describes the standard approach to propositional attitudes, focusing on the analysis of *know* in epistemic logic. Section 8.2 presents the IEL framework, and shows how inquisitive semantics allows us to obtain a more general account of *know* and an account of *wonder*. Finally, Section 8.3 broadens the scope of the discussion, looking at the general view of modal operators that emerges from IEL and sketching some directions for future work.

### 8.1 Propositional attitudes: the standard account

Information-directed attitudes like *know*, *believe*, and *remember* are usually analysed as relations between agents and classical propositions, and referred to as *propositional attitudes*. The traditional analysis of such attitudes goes back to Hintikka (1969), and it lies at the heart of the framework of epistemic logic.

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1 Following the standard linguistic notation, we indicate the ungrammaticality of a sentence by marking it with a ‘*’.
Focusing on the case of knowledge, the standard approach can be described succinctly as follows. For each agent \( a \) we consider a map \( \sigma_a \) which assigns to each possible world \( w \) a set \( \sigma_a(w) \) of possible worlds—those worlds which are compatible with what the agent knows at \( w \). The set of worlds \( \sigma_a(w) \) is referred to as the agent’s epistemic state in the world \( w \).  

In the logical language, one can then add a modal operator \( K_a \) that allows us to talk about what the agent \( a \) knows. A formula \( K_a \varphi \) is true at a world \( w \) if it follows from what \( a \) knows at \( w \) that \( \varphi \) is true. More formally, \( K_a \varphi \) is assigned the following truth-conditions, where \( |\varphi| \) denotes the set of possible worlds where \( \varphi \) is true.

\[
(3) \quad w \models K_a \varphi \iff \sigma_a(w) \subseteq |\varphi|
\]

This logical analysis is not only used in contexts where we want to model and reason about information, which is often the case in logic, economics, artificial intelligence and computer science; it is also standardly assumed in linguistics to be at the core of the workings of the verb *know* and its cross-linguistic kin.

Other information-directed attitudes like *believe* and *remember* can be analyzed in much the same way: in this case, the set \( \sigma_a(w) \) will consist of those worlds that are compatible with what the agent
believes/remembers, and a corresponding modality can be added to
the logical language, which will be interpreted by means of a clause
analogous to (3).
Notice that, from a formal point of view, $K_a$ operates by comparing
two sets of worlds—two propositions in the classical sense: the epistemic
state of the agent, and the proposition expressed by the complement. We
will see in Section 8.3 that, in the inquisitive setting, modalities have
essentially the same behavior, but they will compare two propositions
in the inquisitive sense: both the state of an agent and the proposition
expressed by the complement will be modeled as sets of information
states, encoding both information and issues.

Before turning to the inquisitive take on modality, however, we will
discuss some of the limitations of the standard view on propositional
attitudes and attitude verbs which are most relevant for our purposes.

**Limitation 1: know + interrogatives.** Some verbs expressing information-
directed attitudes, like *know* and *remember*, can combine not only
with declarative complement clauses, but also with interrogative ones.
Focusing on the case of *know*, in English we find not only sentences
like (4), which can be analysed directly by means of clause (3), but also
sentences like (5a–c).

(4) Alice knows that Bob is coming.

(5) a. Alice knows whether Bob is coming.

b. Alice knows whether Bob or Charlie is coming.

c. Alice knows who is coming.

In order to account for the semantics of (5a–c) while maintaining
that *know* primarily operates on a classical proposition, two types
of approaches have been pursued. Groenendijk and Stokhof (1984)
proposed a uniform approach: in their theory, complement clauses,
whether declarative or interrogative, have the same type of denotation—
a classical proposition; in the case of a declarative clause *that* $\alpha$, the
denotation is the set of worlds where $\alpha$ is true, $|\alpha|$; in the case of an
interrogative clause $\mu$, the denotation is the true complete answer to
$\mu$ at the world of evaluation. In either case, the attitude verb can then
apply directly to the denotation of the embedded clause according to
the clause in (3).

By contrast, Karttunen (1977) proposed an approach based on type-
shifting: on this approach, an interrogative clause denotes a set $Q$ of
propositions—the set of true answers to the interrogative. When *know*
combines with such a clause, we proceed in two steps: first, from the denotation $Q$ of the embedded interrogative we derive the classical proposition $\bigcap Q$, which represents the complete answer to $Q$ at the world of evaluation as given by Karttunen's theory; second, an analysis of know in line with (3) is applied to the resulting classical proposition.\(^5\)

Both these approaches rest on the assumption that sentences like (5a–b) always express relations between an individual and a specific classical proposition—the complete answer to the embedded question. But this is problematic. To see why, consider a mention-some question like (6).

(6) Where can one buy an Italian newspaper in Amsterdam?

This question can be completely resolved by providing only one of multiple places where Italian newspapers are sold. Now consider:

(7) Alice knows where one can buy an Italian newspaper in Amsterdam.

There is no single classical proposition that Alice needs to know in order for (7) to be true; rather, (7) may be true by virtue of Alice knowing one among various classical propositions: that one can buy an Italian newspaper at Central Station, that one can buy an Italian newspaper at the airport, etc. Thus, the approaches of Groenendijk and Stokhof (1984) and Karttunen (1977)—which would interpret sentence (7) as a claim that Alice knows a certain classical proposition—cannot assign the right truth-conditions in this case.\(^6\)

In the next section, we will develop an inquisitive counterpart of the epistemic logic account of knowledge. On this account, the attitude of knowing relates an agent to a proposition in the inquisitive semantics sense. Since in inquisitive semantics both declaratives and interrogatives express propositions, we can analyse (4) and (5) in a uniform way, without assuming any type-shifting and without relying on the existence of a complete true answer to the embedded question.

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\(^5\) In addition to this, Karttunen (1977) and Groenendijk and Stokhof (1984) also differ in their notion of complete answer to a wh-question. In the subsequent literature, the two notions have come to be referred to, respectively, as the weakly exhaustive and the strongly exhaustive answer to the wh-question. This difference is orthogonal to our main concerns in this chapter.

\(^6\) To deal with these cases, Groenendijk and Stokhof (1984) propose to treat (6) as having a higher semantic type and to type-shift the entry for the embedding verb so that it can apply to objects of this type. We will not discuss this option in detail here. A problem with this lifting strategy is pointed out in footnote 20 of Ciardelli (2017b).
Limitation 2: issue-directed attitudes  Besides information-directed attitudes like know and believe, there are also issue-directed attitudes like wonder and be curious. Such issue-directed attitudes play a key role in cognition and inquiry (see Friedman, 2013). However, these attitudes cannot be construed at attitudes towards classical propositions: if I wonder who is coming for dinner, the object of my wondering is not a specific classical proposition, but rather the issue as such. For this reason, issue-directed attitudes fall squarely beyond the scope of the traditional approach to propositional attitudes.

These attitudes are also associated with corresponding verbs/constructions in English. Interestingly, these verbs can only take an interrogative complement. For instance, (8a) is grammatical, but (8b) is not.

(8)  a. Alice wonders who is coming for dinner.
    b. *Alice wonders that Bill is coming for dinner.

In the literature, a verb like wonder has mostly been treated as an un-analysed relation between individuals and question meanings (e.g., see Groenendijk and Stokhof, 1984). However, without some analysis of this relation, we lack an account of the entailments licensed by sentences involving wonder. We cannot predict, for instance, that the conclusion in (9c) follows from the premises in (9a) and (9b), but not from either premise alone.

(9)  a. Alice wonders who the culprit is.
    b. Alice knows that the culprit is Bob or Charlie.
    c. So, Alice wonders whether the culprit is Bob or Charlie.

As we will see, in the inquisitive setting issue-directed attitudes can be analysed as relations between agents and inquisitive propositions. The crucial difference between information-directed attitudes and issue-directed attitudes is that, in order to analyse the latter, it is not sufficient to equip an agent with an information state. Rather, we need to be able to represent an agent’s inquisitive state—encoding the issues that the agent is interested in.7

7 A particularly interesting inquisitive attitude is caring. Although the verb care does embed both declaratives and interrogative complements, the attitude of caring itself cannot in general be viewed as having a classical proposition as its object, but should rather be seen as oriented towards an issue. See Elliott et al. (2017) and Ciardelli and Roelofsen (2018) for discussion.
This inquisitive analysis of *wondering* as an attitude also suggests an analysis of the corresponding verb in natural language. As we will see, this analysis accounts for the validity of entailments such as the one in (9), and provides an explanation for the fact that *wonder* does not embed declarative complements.

### 8.2 Inquisitive epistemic logic

In this section, we illustrate the inquisitive approach to propositional attitudes by presenting the framework of *inquisitive epistemic logic* (IEL, Ciardelli and Roelofsen, 2015), which is designed to model not only the knowledge that certain agents have, but also the issues that they are interested in. This framework also provides an inquisitive analysis of *know* and *wonder* that addresses the limitations pointed out in the previous section.

#### 8.2.1 Inquisitive epistemic models

In standard epistemic logic, every agent $a$ is equipped with a map $\sigma_a$, which gives for every possible world $w$ a description of the epistemic state of the agent at $w$, modeled as an information state $\sigma_a(w)$. In *inquisitive* epistemic logic, we model not only the information that an agent has, but also the issues that she entertains. This is done by means of a map $\Sigma_a$ which gives, for every possible world $w$, an issue $\Sigma_a(w)$ over the information state $\sigma_a(w)$, called the *inquisitive state* of agent $a$ at $w$. Intuitively, an information state $s \subseteq \sigma_a(w)$ is in $\Sigma_a(w)$ if and only if all the issues that the agent entertains at $w$ are resolved in $s$. In other words, the states $s \in \Sigma_a(w)$ are those that contain enough information to satisfy the agent’s curiosity. We may view them as the states that the agent would like to reach through inquiry, but one should not read too much into this characterization: in particular, reaching a state in $\Sigma_a(w)$ need not be *desirable* for the agent in an absolute sense (the agent’s issues may well be resolved in ways that the agent finds quite unpleasant).

The constraint that $\Sigma_a(w)$ be an issue over the information state $\sigma_a(w)$, i.e., that $\bigcup \Sigma_a(w) = \sigma_a(w)$, can be viewed as resulting from two requirements. In one direction, we model an agent’s inquisitive state by specifying which enhancements of the agent’s epistemic state contain enough information to resolve the agent’s issues. This means that every $s \in \Sigma_a(w)$ must be a subset of $\sigma_a(w)$, which implies $\bigcup \Sigma_a(w) \subseteq \sigma_a(w)$. For the converse, recall from Section 2.3 that the information state
∪ \Sigma_a(w) captures the information assumed by the issue \Sigma_a(w), i.e., the information needed to guarantee that \Sigma_a(w) can be truthfully resolved. For example, if \Sigma_a(w) is the issue of what Alice’s dog is called, then ∪ \Sigma_a(w) is the information that Alice has a dog. By requiring that \sigma_a(w) ⊆ ∪ \Sigma_a(w), we capture the idea that having the relevant information is a prerequisite for entertaining the issue.8

Now, since \sigma_a(w) = ∪ \Sigma_a(w), the agent’s epistemic state, \sigma_a(w) can always be retrieved from her inquisitive state, \Sigma_a(w). Thus, in effect, \Sigma_a(w) encodes both the knowledge and the issues of agent a in world w. This means that the map \Sigma_a suffices as a specification of the state of the agent at each world, and we do not have to list \sigma_a explicitly as an independent component of our models. This leads to the following definition of inquisitive epistemic models.

**Definition 8.1 (Inquisitive epistemic models)**
A first-order inquisitive epistemic model for a set of agents A is a quadruple M = ⟨W, D, I, ΣA⟩, where:

- ⟨W, D, I⟩ is a first-order information model, in the sense of Definition 4.1.
- ΣA = {Σa | a ∈ A} is a set of state maps Σa, one for each agent a ∈ A, each of which assigns to any world w an issue Σa(w).

We refer to Σa(w) as the inquisitive state of a at w. Moreover, we let \sigma_a(w) := ∪ Σa(w), and we refer to \sigma_a(w) as the epistemic state of a at w.

Just like in standard epistemic logic, this general characterization of inquisitive epistemic models may be supplemented with certain constraints on the agents’ information states and inquisitive states. For instance, the following conditions may (but need not) be imposed on an inquisitive epistemic model:

- **Factivity**: for any w ∈ W, w ∈ \sigma_a(w)
- **Introspection**: for any w, v ∈ W, if v ∈ \sigma_a(w), then \Sigma_a(v) = \Sigma_a(w)

The factivity condition, which is exactly as in standard epistemic logic, requires that the agent’s knowledge be truthful. The introspection condition requires that the agent knows exactly what her state is, with regard

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8 In an enrichment of IEL which also models defeasible beliefs, these requirements may be weakened. For instance, Ciardelli and Roelofsen (2014) propose a framework in which we can distinguish between the agent’s prior issues and the agent’s current issues—the issues entertained given the agent’s current beliefs.
to both knowledge and issues: according to this condition, only worlds where the state of the agent is the same as in w can be compatible with what the agent knows at w; that means that an agent can never be uncertain about what her own inquisitive state is.

These conditions are intended here just as an illustration: the choice of the particular conditions to be imposed on the state maps $\Sigma_a$ will depend on the particular intended application of the framework, and in any case, it is orthogonal to the main novelties introduced by IEL.

8.2.2 Knowledge

To obtain an inquisitive account of knowledge, we extend the first-order language of InqB by means of modal operators $K_a$, where $a$ is the index for an agent, which can be applied without restrictions to any formula in the language.

The semantics of InqB needs to be extended with an inductive clause that specifies how a formula of the form $K_a \varphi$ is to be interpreted. In Chapter 4, we saw that in the inquisitive setting, a semantics can be specified recursively in two ways: one can directly define the proposition $[\varphi]$ expressed by a formula $\varphi$, or one can define a relation of support $s \models \varphi$ between information states and formulas, and then define the proposition expressed by $\varphi$ as the set of states that support $\varphi$: $[\varphi] = \{s \mid s \models \varphi\}$.

In the present context, the latter presentation will be more convenient. We will augment the recursive support clauses given by Fact 4.8 with the following inductive clause for $K_a \varphi$:

$$s \models K_a \varphi \iff \forall w \in s : \sigma_a(w) \models \varphi$$

Notice that this clause ensures that the interpretation of $K_a \varphi$ is persistent: if $s \models K_a \varphi$ and $t \subseteq s$, then $t \models K_a \varphi$. Moreover, $K_a \varphi$ is vacuously supported by the empty information state. This guarantees that the set $[K_a \varphi]$ of supporting states is indeed a proposition in the inquisitive sense—a non-empty and downward closed set of information states—and thus that the system we are defining fits within the general inquisitive semantics framework defined in Chapter 2.

To understand the clause, it is useful to look at the truth-conditions to which it gives rise. Recall that we say that a formula $\varphi$ is true at a world $w$ in case $w \in \text{info}(\varphi)$. Also, recall from Fact 2.14 that truth relative to a world $w$ always amounts to support at the singleton state $\{w\}$. By specializing the support clause in (10) to a singleton state $\{w\}$, we obtain the following truth-conditions for a formula $K_a \varphi$. 
Given these truth conditions, it becomes clear that $K_a \phi$ is supported by a state $s$ just in case it is true at any world in $s$. This means, by Fact 2.19 on page 25, that modal formulas are always non-inquisitive.

**Fact 8.2** For any $\phi$, $K_a \phi$ is non-inquisitive.

This also means that, in order to understand the semantics of $K_a \phi$, we just need to understand at which worlds $K_a \phi$ is true. The set $|K_a \phi|$ of these worlds will be the unique alternative in the proposition expressed by $K_a \phi$.

According to clause (11), the truth-conditions of $K_a \phi$ are very simple: $K_a \phi$ is true at $w$ just in case $\phi$ is supported by the epistemic state of $a$ at $w$. To see what this predicts for some particular cases, consider again the examples in (4) and (5). We assume that these sentences can be translated to our logical language by applying the operator $K_a$ to the translations of the embedded clauses, which we assume to be identical to the translation of the corresponding main clause. Given the translations of these main clauses suggested in Chapter 5, this leads to the following translations:

\begin{enumerate}
  \item a. Alice knows that Bob is coming. \hspace{1cm} K_a Cb
  \item b. Alice knows whether Bob is coming. \hspace{1cm} K_a ?Cb
  \item c. Alice knows whether Bob or Charlie is coming. \hspace{1cm} K_a (Cb \lor Cc)
  \item d. Alice knows who is coming. \hspace{1cm} K_a (\forall x ?Cx)
\end{enumerate}

The truth-conditions for these modal formulas are as follows:

\begin{enumerate}
  \item a. $w \models K_a Cb \iff \sigma_a (w) \subseteq |Cb|
  \item b. $w \models K_a ?Cb \iff \sigma_a (w) \subseteq |Cb|$ or $\sigma_a (w) \subseteq |\neg Cb|
  \item c. $w \models K_a (Cb \lor Cc) \iff \sigma_a (w) \subseteq |Cb|$ or $\sigma_a (w) \subseteq |Cc|
  \item d. $w \models K_a (\forall x ?Cx) \iff \forall d \in D: \sigma_a (w) \subseteq |Cd|$ or $\sigma_a (w) \subseteq |\neg Cd|
\end{enumerate}

For sentence (12a), translated as $K_a Cb$, we obtain the same predictions that standard epistemic logic would deliver: $K_a Cb$ is true if Alice's knowledge implies that Bob is coming. This holds whenever the complement of $K_a$ is non-inquisitive.

**Fact 8.3** If $\phi$ is non-inquisitive, then $w \models K_a \phi \iff \sigma_a (w) \subseteq |\phi|$.

Recall from Chapter 6 that we assume that declarative clauses always involve a projection operator ‘!’ which makes them non-inquisitive.
Thus, we obtain generally that, for natural language sentences in which know embeds a declarative clause, our account coincides with the one given by standard epistemic logic: a sentence Alice knows that $S$ is predicted to be true in case it follows from Alice's knowledge that $S$ is true.

Now, however, the same clause for $K$ can be applied directly to analyze sentences (12b)–(12d), in which know embeds an interrogative clause. The predictions are the expected ones: (12b) is true in case Alice's knowledge implies that Bob is coming, or it implies that Bob is not coming; (12c) is true in case Alice's knowledge implies that Bob is coming, or it implies that Charlie is coming; finally, (12d) is true in case for each individual $d$ in the domain, Alice's knowledge implies either that $d$ is coming, or that $d$ is not coming; in other words, (12d) is true if Alice's knowledge determines exactly what is the set of individuals who are coming.

Notice that this treatment of interrogatives embedded under know, unlike the ones of Groenendijk and Stokhof (1984) and Karttunen (1977), has no problems dealing with knowledge of mention-some questions. To see this, consider again the mention-some question in example (6), repeated below as (47).

(47) Where can one buy an Italian newspaper?  $\exists x.Ix$

If $Ix$ stands for ‘$x$ is a place where one can buy an Italian newspaper,’ then (47) is naturally translated as $\exists x.Ix$: it expresses an issue which is resolved in a state $s$ just in case $s$ implies of some $d$ that $d$ is a place where one can buy an Italian newspaper (see Chapter 5).

(48) $s \models \exists x.Ix \iff \exists d \in D : s \subseteq |Id|$

As a consequence, the statement in (7), repeated here as (49), will be translated as $K_a(\exists x.Ix)$.

(49) Alice knows where one can buy an Italian newspaper. $K_a(\exists x.Ix)$

This yields the following truth-conditions: (49) is predicted to be true in case for some $d$, it follows from Alice’s knowledge that $d$ is a place where one can buy an Italian newspaper.

(50) $w \models K_a(\exists x.Ix) \iff \exists d \in D : \sigma_a(w) \subseteq |Id|$

Thus, we correctly capture that (49) may be true by virtue of Alice knowing different things: in one case, it may be true because she knows
that one can buy an Italian newspaper at Central Station; in another case, it might be true because she knows that one can buy an Italian newspaper at the airport. The problem that we pointed out for the analyses of Groenendijk and Stokhof (1984) and Karttunen (1977) is avoided, because we do not try to reduce (16) to a claim that there is a specific piece of information that Alice knows.

As a last example illustrating the generality of the inquisitive account of knowledge, consider (18).

(18) Alice knows whether Bob will win if he puts down his Ace.

In this case, the embedded clause is a conditional question $Pb \rightarrow ?Wb$, and so (18) as a whole is translated as $K_a(Pb \rightarrow ?Wb)$. This predicts the following truth-conditions for (18):

\[ w \models K_a(Pb \rightarrow ?Wb) \iff \sigma_a(w) \cap |Pb| \subseteq |Wb| \text{ or } \sigma_a(w) \cap |Pb| \subseteq |\neg Wb| \]

That is, (18) is predicted to be true if Alice’s knowledge, restricted to those worlds where Bob puts down his Ace, settles the question whether Bill will win or not. This is the desired prediction, and it is another case where the inquisitive account goes beyond what could be predicted by existing theories of question embedding.\(^9\)

This illustrates how, in inquisitive epistemic logic, the standard analysis of knowledge generalizes smoothly to the case in which the prejacent has non-trivial inquisitive content. This makes it possible to obtain a general account of the verb *know* in combination with both declarative and interrogative clauses—an account that does not require any type-shifting to take place and which extends the empirical coverage of existing accounts—dealing smoothly, among other things, with mention-some questions and conditional questions.

\(^9\) Isaacs and Rawlins (2008), who are specifically concerned with conditional questions, do provide an analysis of such questions embedded under *know*. However, since their analysis of conditional questions is based on discourse dynamics, it requires a significant complication of the lexical entry for *know*. In addition, since their analysis of questions is based on binary relations, their account is less general (due to the limitations discussed in Section 9.3 below). For example, it cannot account for an embedded mention-some conditional question like the one in (i).

(i) Alice knows where Bob can buy an Italian newspaper if he is in Amsterdam.

Finally, essentially due to a technical problem pointed out in footnote 8 of Sano and Hara (2014), Isaacs and Rawlins wrongly predict that knowing a conditional question implies knowing whether the antecedent is true.
8.2.3 Wondering

To provide an analysis of the attitude of wondering, the language of inquisitive epistemic logic is equipped with a second kind of modal operator. Besides the modality $K_a$, for each agent $a$ we also have a modality $E_a$ which allows us to talk about the issues that the agent entertains (whence the notation $E_a$).\(^{10}\) A modal formula $E_a\phi$ is interpreted by means of the following support clause.

\[(20) \ s \models E_a\phi \iff \forall w \in s, \forall t \in \Sigma_a(w) : t \models \phi\]

As in the case of $K_a\phi$, it is easy to see that the support conditions for $E_a\phi$ are persistent and vacuously satisfied by the empty information state, which ensures that $[E_a\phi]$ is a proposition in the inquisitive semantics sense. By specializing the support condition to the case of a singleton state $\{w\}$, we obtain the following truth-conditions for $E_a\phi$.

\[(21) \ w \models E_a\phi \iff \forall t \in \Sigma_a(w) : t \models \phi\]

Given these truth conditions, it is clear that $E_a\phi$ is supported by a state $s$ just in case it is true at any world in $s$. Thus, just like $K_a\phi$, also $E_a\phi$ is always non-inquisitive, regardless of whether $\phi$ is inquisitive or not.

**Fact 8.4** For any $\phi$, $E_a\phi$ is non-inquisitive.

This means that, in order to understand the semantics of $E_a\phi$, we just need to understand at which worlds this formula is true. The proposition expressed by $E_a\phi$ will then have a unique alternative, namely, the set $|E_a\phi|$ of all worlds where the sentence is true.

According to (21), $E_a\phi$ is true at a world $w$ in case $\phi$ is supported by all elements $s \in \Sigma_a(w)$ of the agent’s inquisitive state. Now, recall that the states $t \in \Sigma_a(w)$ are precisely those states where the issues that $a$ entertains at $w$ are resolved. Thus, $E_a\phi$ is true in case, if the issues that $a$ entertains were resolved, $\phi$ would be supported.

Notice that there is a trivial way in which $E_a\phi$ may be true: $\phi$ might already be supported by the current epistemic state of the agent, $\sigma_a(w)$. In this case, since all elements of $\Sigma_a(w)$ are enhancements of $\sigma_a(w)$ and support is persistent, all these elements will support $\phi$ as well, and $E_a\phi$ will be true at $w$. Now, this trivial case holds precisely when $K_a\phi$ is true. This allows us to define a new formula $W_a\phi$ which says that $E_a\phi$ holds non-trivially.

\(^{10}\) Although we read $E_a$ as ‘entertain’—for lack of a better term—we do not propose to regard the modality $E_a$ as an analysis of the verb entertain (or any other English verb).
(22) \( W_a \phi := \neg K_a \phi \land E_a \phi \)

The formula \( W_a \phi \) is true at \( w \) in case (i) the current epistemic state of the agent does not support \( \phi \), but (ii) if the agent’s issues were resolved, \( \phi \) would come to be supported. We may read this less formally as: the agent epistemic state does not support \( \phi \), but the agent strives to reach a state where \( \phi \) is resolved.

The operator \( W_a \) gives us a reasonable analysis of the issue-directed attitude of wondering, and of the corresponding verb in natural language. To see what this analysis predicts, consider (23). By analysing the verb wonder as \( W_a \) and the embedded interrogative in the usual way, we obtain the translation \( W_a ? Cb \).

(23) Alice wonders whether Bob is coming. \( W_a ? Cb \)

Let us consider the truth-conditions that are predicted: \( W_a ? Cb \) is true at a world \( w \) in case:

- Alice’s epistemic state \( \sigma_a(w) \) does not support \( Cb \), that is, \( \sigma_a(w) \) contains both \( Cb \)-worlds and \( \neg Cb \)-worlds;
- all the states \( s \in \Sigma_a(w) \) support \( Cb \), that is, they consist either exclusively of \( Cb \)-worlds, or exclusively of \( \neg Cb \)-worlds.

Condition (i) means that Alice’s current knowledge does not determine whether Bob is coming; condition (ii) means that resolving Alice’s issues is bound to lead to a state that determines whether Bob is coming; in other words, Alice’s issues are not resolved unless it is established whether Bob is coming or not. This sounds like a reasonable analysis of the truth-conditions of (23).

As another example, consider (24), translated as \( W_a (\forall x ? Cx) \).

(24) Alice wonders who is coming. \( W_a (\forall x ? Cx) \)

This sentence is true in case Alice’s current knowledge does not determine exactly which individuals are coming, but in order to resolve Alice’s issues it would be necessary to establish exactly which individuals are coming.

This analysis of wonder provides us with an account of the validities of inferences such as (9), repeated below as (25), as the reader is asked to show in Exercise 8.5.

(25) a. Alice wonders who the culprit is.
   b. Alice knows that the culprit is Bob or Charlie.
   c. So, Alice wonders whether the culprit is Bob or Charlie.
Moreover, this account of wonder also suggests an explanation of the ungrammaticality of sentences like (26), in which wonder embeds a declarative clause.

(26) *Alice wonders that Bob is coming.

According to our analysis, this sentence would be translated as $W_a Cb$.

When is this formula true? By definition, $W_a Cb$ amounts to the conjunction $\neg K_a Cb \land E_a Cb$. Now let us consider what each conjunct requires:

(27) a. $w \models \neg K_a Cb \iff w \not\models K_a Cb \iff \sigma_a \not\subseteq |Cb|

b. $w \models E_a Cb \iff \forall s \in \Sigma_a (w) : s \subseteq |Cb|

These two conditions are contradictory: (27b) implies $\bigcup \Sigma_a (w) \subseteq |Cb|$, that is, $\sigma_a (w) \subseteq |Cb|$, which contradicts (27a). Hence, $W_a Cb$ is a contradiction.11

Notice that, in deriving this result, we have not used anything specific about the embedded clause other than the fact that it expresses a non-inquisitive proposition—a fact which is common to all declarative complements. This means that combining wonder with a declarative complement systematically results in a contradiction. This can be taken to explain the ungrammaticality of this sort of construction (for the connections between systematic contradictions and ungrammaticality, see Gajewski, 2002; Chierchia, 2013; Abrusán, 2014).12,13

To conclude this brief exposition of IEL, let us illustrate the workings of the modal operators of IEL with an example. Figures 8.1(a)-8.1(c) represent the inquisitive states of three agents at a given world $w$. In each case, the solid blocks represent the maximal elements of the agent’s inquisitive state, while the dashed area—corresponding to the union of these blocks—represents the agent’s knowledge. Formally, the agents’
Figure 8.1 The inquisitive states of three agents and the alternatives for \( ?p \).

inquisitive states at the given world are as follows, where \( S^\downarrow \) denotes the downward closure of the set of states \( S \):

- \( \Sigma_a(w) = \{\{11\}, \{10\}\}^\downarrow \)
- \( \Sigma_b(w) = \{\{11, 10\}, \{01, 00\}\}^\downarrow \)
- \( \Sigma_c(w) = \{\{11, 10, 01\}, \{00\}\}^\downarrow \)

As a consequence, the agents’ epistemic states are as follows:

- \( \sigma_a(w) = \{11, 10\} \)
- \( \sigma_b(w) = \sigma_c(w) = \{11, 10, 01, 00\} \)

We take the name of each world to reflect the truth value of two atomic sentences \( p \) and \( q \): at world 11 both are true, at world 10 only \( p \) is true, and so on. The alternatives for the polar question \( ?p \) are depicted in Figure 8.1(d).

Our three agents each stand in a different relation to the question \( ?p \). Alice’s epistemic state implies that \( p \) is true and thereby supports the question: thus, \( K_a?p \) is true, i.e., Alice knows whether \( p \). From this it follows that \( W_a?p \) is false (because the condition \( \neg K_a?p \) fails), i.e., Alice does not wonder whether \( p \).

Bob’s epistemic state is trivial, and it does not support the question \( ?p \). Thus, \( K_b?p \) is false, i.e., Bob does not know whether \( p \). On the other hand, the information states where Bob’s issues are resolved—the elements of \( \Sigma_b(w) \)—are precisely the information states in which \( ?p \) is supported. This means that \( E_b?p \) is true. Together with \( \neg K_b?p \), this implies that \( W_b?p \) is true, i.e., Bob wonders whether \( p \).

Charlie’s epistemic state is trivial as well, which means that, like Bob, Charlie does not know whether \( p \). However, some elements of his inquisitive state (e.g., the information state \( \{11, 10, 01\} \)) fail to support the question \( ?p \). This means that \( E_c?p \) is false, and therefore, \( W_c?p \) is false as well: thus, unlike Bob, Charlie does not wonder whether \( p \).
Interestingly, both Alice and Charlie do not wonder whether \( p \), but for different reasons: in Alice's case, it is because she has already resolved the question, while in Charlie's case, it is because he is not interested in resolving it.

### 8.3 Beyond know and wonder

We have focused our attention in this chapter on some specific modal notions, in a particular logical setting. However, we think that the general approach illustrated by IEL is applicable beyond this restricted setting as well, giving rise to a richer view on the linguistic notion of modality in general. We end this chapter with some programmatic remarks on the potential benefits of such an enriched perspective.

In linguistics, modal expressions are standardly viewed as sentential operators that relate the proposition expressed by their argument (their prejacent) to a proposition encoding a set of relevant background assumptions (the modal base). Some modal expressions indicate that the prejacent is consistent with the modal base (possibility modals), while others indicate that the prejacent is entailed by the modal base (necessity modals). The nature of the modal base depends on the particular flavor of the modal expression. For instance, epistemic modals relate their prejacent to a relevant body of information, while deontic modals relate their prejacent to a modal base determined by a relevant set of rules. Finally, modal expressions differ in their grammatical category. Among the most widely investigated kinds of modal expressions are attitude verbs like know, believe, want, and hope, and auxiliary verbs like might, may, must, and should.

Sophisticated theories have been developed to capture the core mechanisms that underlie the linguistic behavior of all these different types of modal expressions in a unified way (see in particular Kratzer 2012 for a collection of influential articles, and Kaufmann and Kaufmann 2015 for a recent survey). However, while the domain that is covered by these theories is indeed impressively broad, the approach taken in inquisitive epistemic logic suggests a substantial further generalization, both of the linguistic notion of modal expressions as such, and of the theories that deal with them.

Namely, rather than construing modal expressions as relating two classical propositions, we may construe them as relating two propositions in the inquisitive sense. For instance, both modalities \( K_a \) and \( E_a \)
of IEL can be seen as expressing relations between an agent’s inquisitive state $\Sigma_a(w)$ and the inquisitive proposition $[\varphi]$ expressed by the prejacent. This becomes evident once their clauses are re-stated as follows:

- $w \models K_a \varphi \iff \bigcup \Sigma_a(w) \in [\varphi]$
- $w \models E_a \varphi \iff \Sigma_a(w) \subseteq [\varphi]$

This shift in perspective broadens our linguistic view on modality in three ways. First, as exemplified in a very concrete way in IEL, the class of modal expressions becomes richer, now also including ones that take inquisitive constructions as their argument. Thus, it becomes possible to pursue a unified account of propositional attitude verbs like know, believe, want, and hope on the one hand, and issue-directed attitude verbs like wonder, be curious, and care on the other. Second, a more fine-grained notion of modal bases becomes available: we can now interpret modal expressions not only in the context of a certain body of information, but also in the context of a relevant background issue. And third, while on the standard account there are only two salient relations between the prejacent and the modal base, i.e., inclusion (entailment) and overlap (consistency), in the inquisitive setting there are many more, due to the fact that inquisitive propositions carry more structure than simple sets of worlds. This allows for a refinement of the standard dichotomy between possibility and necessity modals.

While these remarks are admittedly very programmatic and clearly stand in need of concrete substantiation, the research programme that they suggest seems an exciting one to pursue. The treatment of know and wonder developed in IEL just constitutes the first step in this direction.

8.4 Pointers to further work

For a more detailed presentation of inquisitive epistemic logic, the reader is referred to Ciardelli and Roelofsen (2015) and Ciardelli (2016d). Besides the knowledge and issues of individual agents, these references are also concerned with collective notions of knowledge and issues—in particular, with the common knowledge and common issues which are publicly shared among the group. The modal logic arising from IEL has been investigated and axiomatized in Ciardelli (2014, 2016d). Various extensions and refinements of IEL have been explored in the recent literature as well. Ciardelli and Roelofsen (2015) and van Gessel (2016) equip the IEL framework with a dynamics that models the way in which a multi-agent scenario evolves when a statement is made or a question is asked, generalizing the analysis of public and
private announcements in *dynamic epistemic logic* (van Ditmarsch *et al.*, 2007). The logic of public announcements in the inquisitive setting is axiomatized in Ciardelli (2017a). Ciardelli and Roelofsen (2014) develop a refinement of IEL that does not only deal with ‘hard knowledge’ but also with defeasible beliefs, which may be revised or retracted. This inquisitive belief revision framework can be used to model not just linguistic information exchange, but also other information-related processes such as rational inquiry, where the interplay between issues and beliefs has been argued to play a crucial role (see, e.g., Olsson and Westlund, 2006). In Theiler *et al.* (2016a,b) the linguistic treatment of *know* suggested in IEL is refined in order to capture, among other things, the observation that the truth of a knowledge attribution requires not only that the agent have enough knowledge to resolve the embedded question, but also that she not believe any pieces of information that would falsely resolve the question (Spector, 2005; George, 2013; Cremers and Chemla, 2016). In Theiler *et al.* (2017), IEL is extended with a treatment of *believe*, which accounts for the fact that this epistemic verb, unlike *know*, is neg-raising (e.g., *Alice doesn’t believe that Bill did it* typically leads to the inference that Alice believes that Bill didn’t do it) and for the fact that it does not take interrogative complements (e.g., *Alice believes whether Bill did it* is ungrammatical). Finally, Roelofsen and Uegaki (2016) and Cremers *et al.* (2017a) discuss possible refinements of the IEL treatment of *wonder* and *believe* in order to obtain a more comprehensive account of the ignorance inferences that these verbs trigger. For instance, when *wonder* embeds an alternative question, it does not just imply that the issue expressed by the question is not yet resolved in the subject’s current information state, but also that all the alternatives that make up the issue are still compatible with the subject’s information state (e.g., *Alice wonders whether Bob, Charlie, or Daniel did it* does not only imply that Alice does not know yet who the culprit is, but also that she cannot yet rule out any of Bob, Charlie, and Daniel as a potential culprit).

### 8.5 Exercises

**Exercise 8.1 Truth conditions of knowledge and wonder attributions**

Consider again the situation described in Figure 8.1. For each question $\mu$ in the list below, determine for which agents $x$ the formulas $K_x\mu$ and $W_x\mu$ are true.

- $?q$
- $?(p \land q)$
• ?!(p ∨ q)
• ¬q → ?p

Exercise 8.2 Reasoning about knowledge and wondering

Consider the inference in (9), repeated here as (28):

(28) a. Alice wonders who the culprit is.
    b. Alice knows that the culprit is Bob or Charlie.
    c. So, Alice wonders whether the culprit is Bob or Charlie.

1. Translate the premises and the conclusion of the inference in the language of IEL. Use a predicate \( C \) which stands for 'is the culprit', and assume that this predicate is satisfied by exactly one individual at each world in the model. Recall that embedded declarative clauses are translated as involving a projection operator '!'.

2. Prove that if (28a) and (28b) are true at a world, so is (28c).

3. Prove that (28a) and (28b) entail (28c) in the sense of inquisitive semantics.

Exercise 8.3 Inquisitive epistemic logic

For \( \Box \in \{K_a, E_a, W_a\} \), determine whether the following logical principles are valid or invalid. Provide proofs or counterexamples.

• Normality: \( \Box(\varphi \rightarrow \psi) \rightarrow (\Box \varphi \rightarrow \Box \psi) \)

• Distribution laws:
  - \( \Box(\varphi \land \psi) \leftrightarrow \Box \varphi \land \Box \psi \)
  - \( \Box(\varphi \lor \psi) \leftrightarrow \Box \varphi \lor \Box \psi \)
  - \( \Box(\varphi \lor \psi) \leftrightarrow \Box ! (\Box \varphi \lor \Box \psi) \)

• Monotonicity: if \( \varphi \models \psi \), then \( \Box \varphi \models \Box \psi \)

• Necessitation: if \( \models \varphi \), then \( \models \Box \varphi \)

Exercise 8.4 Ignorance

Consider a new modal operator \( N_a \) in IEL, where \( N_a \varphi \) is informally read as 'a is completely ignorant with regard to \( \varphi \)'. Define a suitable semantic interpretation of \( N_a \varphi \), which ensures that whenever \( \varphi \) is non-inquisitive, \( N_a \varphi \) is equivalent to \( \neg K_a ? \varphi \).
Comparison to alternative approaches

As we have seen in previous chapters, one of the main purposes of inquisitive semantics is to serve as a framework for the semantic analysis of questions in natural languages. In this chapter we will compare inquisitive semantics with some other frameworks which have been proposed for this purpose, and which have been used widely in the literature. In doing so, we will restrict our attention to those previous proposals that are most closely related to our own. That is, we will consider the alternative semantics framework proposed by Hamblin (1973) and Karttunen (1977), the partition semantics developed by Groenendijk and Stokhof (1984) and later cast in a dynamic framework by Jäger (1996), Hulstijn (1997), and Groenendijk (1999), and the inquisitive indifference semantics proposed by Groenendijk (2009) and Mascarenhas (2009). We will argue that the framework presented here preserves the essential insights that have emerged from these previous approaches, while overcoming their main shortcomings.1

Figure 9.1 provides a global overview of the different approaches. In this figure, the proposed frameworks are ordered from left to right in terms of restrictiveness—i.e., the constraints they impose on what qualifies as a suitable question meaning—and the development through time is indicated by the bent arrows. Alternative semantics, developed in the 1970s, is the least restrictive among these framework, i.e., the one that imposes the least constraints on what qualifies as a suitable question meaning. This, as we will see below, leads to problems

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1 One prominent approach that we will not discuss here is the functional approach (sometimes also called the categorial or the structured meanings approach), which has its roots in the work of Hull (1975), Tichý (1978), and Hausser and Zaefferer (1978), and has been further developed by Ginzburg and Sag (2000), Krifka (2001a), and Ginzburg (2005), among others. For overviews of the literature on questions, we refer to Groenendijk and Stokhof (1997), Ginzburg (2010), Krifka (2011), Cross and Roelofsen (2014), Dayal (2016), and Dekker et al. (2016).

of overgeneration. Partition semantics on the other hand, originally proposed in the 1980s and further developed in a dynamic setting in the 1990s, is the most restrictive framework—leading to problems of undergeneration. More recent work has tried to strike an optimal balance between these two extremes, first leading to inquisitive indifference semantics (Groenendijk, 2009; Mascarenhas, 2009) and then to the present inquisitive semantics framework.

Alternative semantics will be discussed in Section 9.1, partition semantics in Section 9.2, and inquisitive indifference semantics in Section 9.3, drawing comparisons in each case with the inquisitive semantics framework presented in this book. After having examined and compared these different frameworks for question semantics in some detail, we shift our attention in Section 9.4 to another fundamental issue, concerning the division of labor between question semantics and other components of a general theory of the interpretation of questions, including a theory of speech acts and discourse pragmatics. What exactly should the role of a compositional semantic theory of questions be within such a larger theory of interpretation? We will first consider the received view on this issue, and then compare it to the one taken in inquisitive semantics, which we argue to be more parsimonious.

9.1 Alternative semantics

Alternative semantics was first proposed by Hamblin (1973), driven by the following idea:

Questions set up a choice-situation between a set of propositions, namely those propositions that count as answers to it. (Hamblin, 1973, p. 48)

Thus, Hamblin takes questions to denote sets of classical propositions. These propositions are often referred to as alternatives, hence the name

![Figure 9.1: Semantic frameworks for the analysis of questions, ordered chronologically and in terms of restrictiveness.](image-url)
of the framework. Karttunen (1977) independently proposed a very similar view on question meanings: he also took questions to denote sets of classical propositions, though he restricted the denotation of a question in a particular world to propositions that correspond to answers that are true in that world. In both systems, the meaning of a question, i.e., its intension, is a function from worlds to sets of classical propositions. In Hamblin’s system, this function maps every possible world to the same set of propositions, corresponding to the set of all possible answers; in Karttunen’s system, every world is mapped to a subset of all possible answers, namely those that are true in the given world. As noted by Karttunen (1977, p. 10), this difference is inessential. In both cases, the meaning of a question is fully determined by—and could be identified with—the set of all classical propositions that correspond to a possible answer.2

This classical view on question meanings faces some fundamental problems. We will discuss these, and show that they no longer arise in inquisitive semantics.

9.1.1 First problem: Possible answers

The first problem is that the framework’s core notion—that of a possible answer—is difficult to pin down. Surely, Hamblin and Karttunen provide a compositional semantics for a fragment of English, and thereby specify what they take to be the possible answers to the questions in that fragment. But in order to assess such a compositional theory, or even to properly understand what its predictions amount to, we first need to have a pre-theoretical notion of possible answers, one that the theoretical predictions can be evaluated against. The problem is that such a pre-theoretical notion is difficult, if not impossible to identify. To illustrate this, consider the question in (1) and the responses in (2):

(1) What is Alice’s phone number?
(2) a. It is 055-9090231.
   b. It is 055-9090231 but she prefers to be contacted by email.

2 It should be noted that there are significant differences between Hamblin’s and Karttunen’s approach concerning the compositional derivation of question meanings. While Karttunen sticks to the standard Montagovian architecture, Hamblin proposes a rather radical departure from it, adapting the semantic type of all lexical items and letting the operation that is standardly used to compose the meanings of two constituents, i.e., function application, operate in a pointwise fashion. This compositional architecture, however, faces a number of thorny problems (see, e.g., Shan, 2004; Novel and Romero, 2010; Charlow, 2014). In inquisitive semantics these problems can be overcome in a principled way. A detailed discussion of compositionality, however, is beyond the scope of this book; we refer to Ciardelli, Roelofsen, and Theiler (2017a).
c. It is the same as Bob’s number but with ‘1’ instead of ‘0’ at the end.
d. It is either 055-9090231 or 055-9090233.
e. It starts with 055-9090.

In principle, each of the classical propositions expressed by the declaratives in (2) could be seen as a possible answer to (1). For Hamblin and Karttunen, only (2a) counts as such. However, it is not clear what the precise criteria are for being considered a possible answer, and on which grounds (2a) is to be distinguished from (2b–e).

In inquisitive semantics, question meanings are also sets of classical propositions, just like in alternative semantics. However, in inquisitive semantics these classical propositions are not thought of as the ‘possible answers’ to the question. Rather, they are thought of as the information states—or equivalently, the pieces of information—that resolve the issue that the question expresses. As a consequence, in inquisitive semantics question meanings cannot be defined as arbitrary sets of classical propositions, which is what Hamblin and Karttunen take them to be. Rather, they have to be downward closed. After all, if an information state \( s \) resolves the issue expressed by a given question \( Q \), then any stronger information state \( t \subseteq s \) will also resolve the issue expressed by \( Q \). As a consequence, inquisitive semantics is more restrictive than alternative semantics.

Unlike the notion of a possible answer, the notion of a resolving information state has a clear pre-theoretical significance. For instance, here are two concrete ways to assess empirically whether an information state \( s \) should count as one in which the issue expressed by a question \( Q \) is resolved. Imagine an agent \( a \) whose information state is \( s \), and suppose that the information available in \( s \) is true:\(^3\)

- **Knowledge test**: would we say that \( a \) knows \( Q \)?
  - If the answer is *yes*, \( s \) should count as a state in which the issue expressed by \( Q \) is resolved.
  - Otherwise, \( s \) should not count as such a state.
- **Wondering test**: is it possible for \( a \) to be wondering about \( Q \)?
  - If the answer is *no*, \( s \) should count as a state in which the issue expressed by \( Q \) is resolved.
  - Otherwise, \( s \) should not count as such a state.

\(^3\) For the knowledge test, readers concerned about Gettier cases should replace the term ‘information state’ by ‘knowledge state’, and assume that only information that properly qualifies as knowledge is reflected in \( s \).
If we apply these tests to the above example, it is clear that the information states corresponding to (2a–b) qualify as resolving states for (1), while those corresponding to (2c–e) do not. Thus, theories of questions formulated in inquisitive semantics can be assessed empirically in a way that theories that yield sets of ‘possible answers’ cannot—at least in the absence of a more precise characterization of what ‘possible answers’ are supposed to be.

Even though inquisitive semantics deliberately does not rely on the notion of ‘possible answers’ as a primitive notion, the framework does of course allow us to define various notions of answerhood. For instance, we may characterize a minimal resolving answer to a question Q as a piece of information that:

(i) resolves the issue expressed by Q, and
(ii) does not provide more information than necessary to do so, i.e., is not strictly stronger than any other piece of information that also resolves the issue expressed by Q.

Under this definition, the minimal resolving answers to Q correspond precisely to what we called the alternatives in the proposition expressed by Q. In the above example, (2a) would count as a minimal resolving answer to the question in (1) under this definition, while (2b–e) would not. This is the same distinction that Hamblin and Karttunen made. Now, however, it is clear on which ground the distinction is made.

4 By the information state corresponding to a declarative sentence we mean the set of worlds where the sentence is true, in an information model containing no background information.

5 Recall from footnote 3 in Chapter 2 that some propositions in InqB do not contain any alternatives. According to the characterization of minimal resolving answers just given, questions expressing such propositions do not have any minimal resolving answers. See Ciardelli (2010), Ciardelli et al. (2013b), and Roelofsen (2013a) for further discussion of such cases.

6 It is important to note that the Hamblin/Karttunen notion of a possible answer does not in general coincide with the notion of a minimal resolving answer. For instance, consider the question (i) under its usual, mention-all reading.

(i) Who passed the exam?
(ii) a. \{w | everybody except for Bob passed the exam in w\}
   b. \{w | Alice passed the exam in w\}

Assuming that it is known which individuals make up the intended domain, the classical proposition in (iia) qualifies as a minimal resolving answer to (i). The one in (iib) does not, since it provides no information on which individuals passed the exam besides Alice. For Hamblin and Karttunen, the situation is reversed: the proposition in (iib) qualifies as a possible answer to (i), but not the one in (iia). Thus, the Hamblin/Karttunen
Besides characterizing minimal resolving answers along these lines, we may also define notions of *partial answerhood* and *subquestionhood* (see Groenendijk and Roelofsen, 2009), which are important for the analysis of discourse structure and information structure (see, e.g., Ginzburg, 1996; Roberts, 1996; Büring, 2003). What is crucial is that in inquisitive semantics the meaning of a question is not characterized *in terms of* possible/minimal/complete/partial answers. Rather, as depicted in Figure 9.2, it is the other way around: question meanings, i.e., issues, are defined in terms of what it takes to resolve them, and the possible/minimal/complete/partial answers to a question are defined in terms of these resolution conditions. As a consequence, whichever notion of answerhood we choose to adopt, there will be no need for such a notion to correspond directly to some pre-theoretical concept. Rather, it will be grounded, in a precisely circumscribed way, in the pre-theoretical notion of what it takes for a given issue to be resolved.7

9.1.2 Second problem: Entailment

A second fundamental problem for alternative semantics, which was pointed out and discussed at length in Groenendijk and Stokhof (1984), is that it is difficult to define a suitable notion of *entailment* in this framework that determines when one question is more demanding than another. 

Figure 9.2: Primitive and derived notions in alternative/inquisitive semantics.

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7 Recall from Section 5.6.2 that the issue expressed by questions in natural language is sometimes not completely determined by linguistic conventions; various contextual factors may play a role as well (the intended domain of quantification, the intended method of identification, the intended level of granularity, and the general decision problem that the speaker aims to resolve in asking the question). The challenges that are involved in modeling this context-sensitivity are orthogonal to the challenge of suitably representing the issue expressed by a question under fixed assumptions about the relevant contextual parameters. Inquisitive semantics, just like the work of Hamblin (1973) and Karttunen (1977), addresses the latter challenge, but remains neutral with respect to the former.
another. One consequence of this is that it is hard, if not impossible, to give a principled account of the interaction between questions and logical connectives and quantifiers. For instance, it proves problematic to give a satisfactory treatment of the conjunction of two questions. Without a suitable notion of entailment, conjunction can certainly no longer be treated as a meet operator.\(^8\)

This problem does not arise in inquisitive semantics, which comes with a natural and well-behaved notion of entailment. As discussed in Chapter 3, the space of propositions in inquisitive semantics, ordered by entailment, has a familiar algebraic structure, and a natural treatment of the logical connectives is obtained by associating them with the basic operations in this algebra. Thus, as we have seen, the classical treatment of conjunction as a meet operation can be preserved in inquisitive semantics to apply to informative and inquisitive sentences in a uniform way, and the same goes for the other operations.

The two problems that we just discussed for alternative semantics are closely related. After all, if it were possible to ground the notion of ‘possible answers’ in some pre-theoretical notion, then it would most likely also become clear how to characterize entailment. That is, if there were clear criteria for what it takes to count as a possible answer, we would also know better on which grounds two sets of possible answers should be compared, and under which conditions one set should be seen as entailing another.\(^9\)

Compare the situation with the one we have in classical logic. There, the proposition expressed by a sentence is a set of possible worlds. These worlds are intended to correspond to situations that are compatible with the information that the sentence conveys. In this case, there is a clear pre-theoretical intuition to build on, as to whether a certain situation is or is not compatible with a given piece of information. As a consequence, it is also clear when one sentence should be taken to entail another, namely if it conveys at least as much information,

\(^8\) See Roelofsen (2013a); Ciardelli and Roelofsen (2017a); Ciardelli et al. (2017a) for more elaborate discussion of this point, and a critical assessment of some concrete notions of entailment and conjunction that may be considered in alternative semantics.

\(^9\) It is not the formal notion of meaning as such that stands in the way of a suitable notion of entailment, but really the conception of these meanings in terms of possible answers. For instance, if we construe the meaning of a sentence as a set of classical propositions, as in alternative semantics, but think of these propositions as those that the sentence draws attention to, rather than as possible answers, then it is quite straightforward to define a suitable notion of entailment, which compares two sentences/meanings in terms of their attentional strength (Roelofsen, 2013b).
meaning that the proposition it expresses is a subset of the proposition expressed by the other sentence. In alternative semantics, the meaning of a question is a set of classical propositions which are intended to correspond to its possible answers. However, since it is not clear when exactly a proposition should count as a possible answer, it is also difficult to say when one question should entail another.

In inquisitive semantics, the proposition expressed by a question is a set of information states, which are intended to be those information states that resolve the issue that the question expresses. It is also clear, then, when one question is more demanding than another, namely if every information state that resolves the former also resolves the latter. This immediately delivers the desired notion of entailment, as well as the algebraic operations that are characterized in terms of it.

9.1.3 Third problem: Overgeneration

A third problem, which is again connected to the other two, is that there are question meanings in alternative semantics which seem impossible to express in natural languages. These are question meanings containing two alternatives $\alpha$ and $\beta$ such that one is strictly contained in the other, $\alpha \subset \beta$.

One may think that such meanings may be expressed by disjunctive questions, where each disjunct contributes one of the two alternatives. However, in order to get that $\alpha \subset \beta$, we would have to construct the question in such a way that one disjunct entails the other. As illustrated in (3) and (4) below, such questions are infelicitous (Ciardelli and Roelofsen, 2017a).

(3) #Is John American, or is he Californian?
(4) #Is the value of $x$ different from 6, or is it greater than 6?

It has been well-known since Hurford (1974) that disjunctive declaratives where one disjunct entails the other are generally infelicitous as well.

(5) #John is American or he is Californian.
(6) #The value of $x$ is different from 6 or it is greater than 6.

This phenomenon, known as Hurford’s constraint, has been given an appealing explanation in terms of redundancy. More specifically, Katzir and Singh (2013) propose the following principle (see also Simons, 2001; Meyer, 2014, for closely related proposals):
Local redundancy: a sentence is deviant if its logical form contains a binary operator \( \circ \) applying to two arguments \( A \) and \( B \), and the outcome \( A \circ B \) is semantically equivalent to one of the arguments.10

Let us briefly consider how this principle predicts Hurford’s constraint. In classical semantics, the meaning of a sentence \( A \) is a classical proposition \( |A| \), the set of worlds where the sentence is true. \( A \) entails \( B \) just in case \( |A| \subseteq |B| \). Moreover, sentential disjunction yields the union of two propositions, that is, \( |A \lor B| = |A| \cup |B| \).

Now, suppose that the logical form of a sentence contains a sentential disjunction operator applying to two arguments \( A \) and \( B \) such that \( |A| \subseteq |B| \), as in examples (5) and (6). Then we have that \( |A \lor B| = |A| \cup |B| = |B| \). So, the output is semantically equivalent to one of the inputs. Thus, the given logical form exhibits local redundancy and is therefore predicted to be deviant.

Now, one would of course hope that this explanation of Hurford’s constraint in terms of redundancy would apply not only to declaratives like (5) and (6), but also to questions like (3) and (4). But this is not the case in alternative semantics, where the disjuncts express singleton sets, \( \{|A|\} \) and \( \{|B|\} \), respectively, and disjunction yields the set \( \{|A|, |B|\} \).

Since the output of the disjunction operator is different from any of its inputs, the local redundancy condition is not violated, and no deviance is therefore predicted.

In inquisitive semantics, the explanation of Hurford’s constraint in terms of redundancy does naturally apply to questions like (3) and (4). Assuming that each of the disjuncts expresses a proposition containing all states that consist exclusively of worlds where that disjunct is true (just like atomic sentences in \( \text{InqB} \)), and that \( \lor \) is analysed by means of the inquisitive disjunction operator, we have \( |A| = \wp(|A|) \), \( |B| = \wp(|B|) \), and \( |A \lor B| = |A| \cup |B| = \wp(|A|) \cup \wp(|B|) \). Thus, the output of the disjunction operator is identical to one of its inputs, and redundancy is predicted just as for declarative Hurford disjunctions.11

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10 Katzir and Singh (2013)’s proposal is relativized to a context of utterance \( c \). Since context-dependency plays no role in our discussion, we omit reference to contexts for ease of exposition.

11 It should be noted that there are apparent counterexamples to Hurford’s constraint, which may seem to undermine the argument that we are making here. For instance:

(i) Bill solved two of the homework problems, or he solved all of them.

At first blush, it seems that the second disjunct entails the first, and yet the sentence is felicitous. However, as argued in detail by Chierchia et al. (2009), in such cases the weaker disjunct receives an exhaustive interpretation—here, that Bill solved only two of the problems—which in effect makes it logically independent from the other disjunct. In fact,
Comparison to Alternative Approaches

Let us try to better understand this contrast between inquisitive semantics and alternative semantics by considering the notion of ‘alternatives’ that plays a role in the two frameworks. We have seen that both frameworks associate questions with sets of alternatives, but that the status of these alternatives crucially differs from one framework to the other.

In inquisitive semantics, the alternatives in the proposition expressed by a question are characterized as those pieces of information that resolve the issue that the question raises in a minimal way. This implies that sets of alternatives have to be of a particular form: two alternatives are always logically independent, that is, one is never contained in the other.

In alternative semantics on the other hand, there is no such constraint on sets of alternatives: any set will do. This is connected, of course, to the fact that the notion of an alternative is a primitive notion in this framework, not defined in terms of resolution conditions or any other more elementary notion.

Let us say that a set of classical propositions is non-nested if no proposition in the set is included in another. In inquisitive semantics, then, unlike in alternative semantics, only non-nested sets of classical propositions are regarded as proper sets of alternatives. Thus, certain meanings in alternative semantics do not have a counterpart in inquisitive semantics. It is precisely these additional meanings, i.e., nested sets of alternatives, which seem impossible to express in natural languages. In principle, a Hurford disjunction would be exactly the right kind of construction to express a nested set of alternatives. But we have seen that such disjunctions are infelicitous. This seems to indicate that there is something wrong with nested sets of alternatives as meanings, which is puzzling from the perspective of alternative semantics, since in this framework nested sets of alternatives have exactly the same status as non-nested sets.

In inquisitive semantics, the puzzle does not arise, because nested sets of alternatives simply do not exist. Importantly, such sets are not ruled out by some special purpose constraint: rather, it just follows from the

Hurford’s constraint allows us to explain why the only reading of (i) is one in which the first disjunct receives an exhaustive reading. So, far from undermining the existence of Hurford’s constraint, cases like (i) provide further evidence for it. For a more detailed exposition of the argument that we are presenting here, taking cases like (i) into account, we refer to Ciardelli and Roelofsen (2017a).
way alternatives are construed that they are never nested. This means that from the perspective of inquisitive semantics, what is special about Hurford disjunctions is not that they express some distinguished class of meanings, but rather that they involve redundant disjuncts, which fail to contribute an alternative to the meaning of the disjunction. As we have seen, this is precisely what explains their infelicity.

9.2 Partition semantics

Departing from Hamblin and Karttunen’s work, Groenendijk and Stokhof (1984) propose that a question does not denote a set of classical propositions at each world, but rather a single classical proposition embodying the true exhaustive answer to the question in that world. For instance, if $w$ is a world in which Paul and Nina are coming for dinner, and nobody else is coming, then the denotation of (7) in $w$ is the classical proposition expressed by (8).

(7) Who is coming for dinner tonight?
(8) Only Paul and Nina are coming.

The meaning of a question, i.e., its intension, then amounts to a function from worlds to classical propositions. In Groenendijk and Stokhof’s framework these classical propositions are required to have two special properties: they have to cover the entire logical space (since we must have a true exhaustive answer at every world), and they have to be mutually exclusive (since at each world, only one exhaustive answer can be true). So, in Groenendijk and Stokhof’s theory the meaning of a question is determined by a set $\rho$ of classical propositions that together form a partition of the logical space.

9.2.1 Problem: Undegeneration

Partitions correspond to a specific kind of issues. Indeed, if a given question $Q$ has a true exhaustive answer at each world, then resolving the question amounts to providing an exhaustive answer. This means that if $Q$ is associated with a partition $\rho$, then $Q$ is resolved at a state $s$ if and only if $s$ is included within some complete answer $t \in \rho$. This shows that each partition $\rho$ determines an issue $I_{\rho}$ consisting of all states that are contained in one of the cells of the partition:

$$I_{\rho} := \{s \subseteq t \mid t \in \rho\}$$
However, not every issue corresponds to a partition. More importantly, not all issues expressed by natural language questions correspond to partitions. This is so only for those questions that have an exhaustive answer at every world, in the following sense (for a precise statement of this fact, see Exercise 9.5).

(9) **Exhaustive answer at a world:** we say that a classical proposition $a$ is the exhaustive answer to an issue $I$ at a world $w$ if (i) $a$ is true at $w$ and (ii) for all propositions $p$ true at $w$: $p \in I \iff p \subseteq a$.

There are two different reasons why a question may fail to have an exhaustive answer at a world. First, it may not be possible to truthfully resolve the question at some possible worlds. For instance, consider (10): as soon as our logical space includes worlds where Alice does not have a husband, (10) cannot have an exhaustive answer at these worlds.

(10) How old is Alice’s husband?

However, this limitation can be overcome quite straightforwardly: it suffices to take a question to express a partial function from worlds to exhaustive answers, which corresponds to a partition of a subset of the logical space.

However, the partition theory cannot be patched up in a similar way to deal with questions that allow for various minimal resolving propositions that are all true at some world. An important class of questions with this feature is that of *mention-some* questions like those in (11), which we discussed in Section 5.4.2.

(11) a. What is something that Alice really likes?
   b. Where can I buy an Italian newspaper around here?
   c. What is a typical French dish?
   d. What is an example of an arithmetic theorem which is not provable in Peano Arithmetic?

In order to resolve (11a) it is necessary and sufficient to establish of some $x$ that $x$ is something that Alice really likes. If Alice really likes string quartets as well as scuba diving, then the proposition that she likes string quartets and the proposition that she likes scuba diving are both true at the actual world, and both resolve (11a) in a minimal way. Thus, at the actual world, there is no single proposition that counts as the exhaustive answer to (11a). This implies that the issue expressed by (11a) cannot be represented as a partition.
In addition to mention-some questions, the class of questions which express non-partition issues also includes conditional questions like (12a), disjoined questions like (12b), open disjunctive questions like (12c), and approximate-value questions like (12d) (about the latter type of question, see exercise 9.4).

(12)  
  a. If Alice wins two tickets to Paris, who will she take with her?  
  b. Where can we rent a car, or who has one that we could borrow?  
  c. Does Igor speak Spanish, or French?  
  d. How many stars are there in the Milky Way, give or take ten?

Thus, while partition semantics gives an attractive analysis of questions, which avoids the problems that we highlighted above for alternative semantics, it does so at the cost of significant restrictions on its empirical scope.12

9.2.2 A possible concern: disjunctions of questions

Inquisitive semantics provides a notion of question meaning that is richer than the one assumed in partition semantics, and we have just seen that this is crucial in order to accommodate several classes of questions which express issues that do not correspond to partitions of the logical space. However, this greater generality may also raise a certain concern.

Consider the following sentence from Szabolcsi (1997, p. 325), a disjunction of two wh-questions, which is decidedly odd.

(13)  Who did you marry or where do you live?

Szabolcsi (1997, 2015a) has argued that the oddness of this sentence can be explained in partition semantics. For a partition may be identified with an equivalence relation on the space of possible worlds, and while the intersection of two equivalence relations is itself again an equivalence relation, the same is not true of the union of two equivalence relations. If conjunction and disjunction are taken to express intersection and union, respectively, it is to be expected that conjunction, but not disjunction, can apply to two questions to form a new question in natural languages.

On the other hand, in inquisitive semantics the oddness of (13) cannot be explained on purely semantic grounds, because if we take

12 For a more detailed discussion of the relations between inquisitive semantics and partition semantics, see Ciardelli et al. (2015, §5) and Ciardelli (2017b).
disjunction to express the join operator it delivers a perfectly sensible issue, one that can be resolved either by establishing whom the addressee married or by establishing where the addressee lives. This issue does not correspond to a partition, but it is an issue nonetheless in our framework. Thus, while the inquisitive notion of meaning has important advantages with regard to partition semantics, it may also seem to have a certain disadvantage.

However, note that the prediction arising from partition semantics is a very strong one: it implies that questions cannot be directly disjoined in natural languages at all. Szabolcsi (1997, 2015a) claims that this general prediction is indeed borne out, but we are convinced by examples like (14), repeated from Chapter 1, that it is too strong: disjunctions of questions are not always infelicitous.

(14) Where can we rent a car, or who might have one that we could borrow?

We should note that Szabolcsi remarks that a sentence like (13) may be marginally acceptable if regarded as a case in which the speaker first asks the question who did you marry, but then reconsidered and proposes to replace this first question by the second, where do you live. In such cases, Szabolcsi suggests, disjunction does not play its usual role but is rather used as a corrective device.

Our example (14), however, can be uttered by someone without any reconsideration halfway, and it can be addressed by an addressee as a single question, to which both disjuncts contribute. So, (14) seems to be a genuine disjunction of questions.

Szabolcsi (1997) does not base her empirical claim merely on cases like (13) but also on observations about embedded questions in Hungarian. Hungarian complement clauses, whether declarative or interrogative, are always headed by the subordinating complementizer hogy. Szabolcsi argues that this subordinating complementizer expresses a lifting operation that needs to be invoked before two

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13 Krifka (2001b) endorses Szabolcsi’s claim, though he offers a different explanation, based on the assumption that questions do not express sets of propositions or partitions, but rather speech acts, which Krifka models as operations on commitment states. Speech act disjunction does not exist according to Krifka, because it “would lead to disjunctive sets of commitments, which are difficult to keep track of” (Krifka, 2001b, p. 16).

14 Haida and Repp (2013) also challenge Szabolcsi’s empirical claim, although they maintain a weaker version of it: questions can only be disjoined in downward entailing or non-veridical contexts. Our example (14) presents a challenge for this weaker claim as well.
interrogative complement clauses can be disjoined (just like proper names have to be lifted into generalized quantifiers when they are conjoined or disjoined with a quantificational noun phrase). Support for this idea comes from examples like (15) and (16) below, which indicate that (i) conjoined interrogative complement clauses can have either two occurrences of hogy, applying to both individual conjuncts, or a single occurrence of hogy, applying to the conjunction as a whole, but (ii) disjoined interrogative complement clauses must have two occurrences of hogy, each applying to one of the individual disjuncts.

(15) János megtudta, hogy kit vettél feleségül és Janos found.out subord whom you.took as.wife and (hogy) hol laksz. (subord) where you-live 'Janos found out whom you married and where you live.'

(16) János megtudta, hogy kit vettél feleségül vagy Janos found-out subord whom you.took as.wife or *(hogy) hol laksz. *(subord) where you.live 'Janos found out whom you married or where you live.'

Szabolcsi concludes from this observation that disjunction cannot directly apply to interrogative complement clauses, but always requires intervention of a lifting operation, expressed overtly in Hungarian by hogy.

However, there are counterexamples to the generalization. The Hungarian counterpart of our example (14) is a case in point. When embedded, it may come either with one or with two occurrences of hogy, no matter whether the embedding verb is extensional (e.g., find out) or intensional (e.g., investigate).°  

(17) Péter megtudta, hogy hol tudunk autót bérelní Peter found.out subord where can.we car rent.inf vagy (hogy) kinek van egy, amit kölcsönvehetnénk. or (subord) who.to is one which could.borrow.we 'Peter found out where we can rent a car or who has one that we could borrow.'

° We are grateful to Donka Farkas, Anikó Liptak, and Anna Szabolcsi for discussion of this datapoint.
(18) Péter azt vizsgálja, hogy hol tudnánk autót bérelni vagy (hogy) kinek van egy, amit kölcsönvehetnénk.

'Peter is investigating where we could rent a car or who has one we could borrow.'

In (18), a single occurrence of *hogy* favors a reading on which disjunction takes narrow scope with regard to the verb, while two occurrences of *hogy* favor a reading on which disjunction takes wide scope (Peter is investigating where we can rent a car or he is investigating who has one we could borrow), a pattern that is in line with Szabolcsi’s idea that *hogy* expresses a lifting operation.

It thus seems that, at least in some cases, disjunction *can* apply directly to questions, both in English and in Hungarian. A question that naturally arises, then, is whether the general disjunction operation that inquisitive semantics makes available allows us to derive the correct meaning for those disjunctions of questions which are felicitous. For disjunctions of non-*wh*-questions, we have already argued this to be the case in Chapter 6. The predictions for disjunctions of *wh*-questions also seem to be correct. For instance, assuming that (14) is an open interrogative list and that the two interrogative clauses each receive a mention-some interpretation, the sentence is predicted to express an issue which can be resolved either by identifying a place where the speaker can rent a car, or by identifying a person who might have a car that the speaker can borrow, or by establishing that there is no such place and no such person. These are indeed the resolution conditions we expect for (14). Notice that this prediction is obtained simply by applying inquisitive disjunction to the propositions expressed by the two interrogative clauses—the same disjunction operation that, in Chapter 6, we took to be at work in disjunctive non-*wh*-questions, as well as disjunctive declaratives.

Thus, after all, disjunctive questions seem to provide a strong argument in favor of inquisitive semantics over partition semantics, where examples such as (14) can only be handled at the cost of a significant complication of the framework (and one that gives up some of its most attractive features, such as the general account of entailment and coordination among interrogatives; see Groenendijk and Stokhof, 1989).
Of course, an interesting question that remains to be addressed is why our example (14) behaves so differently from Szabolcsi’s example (13), both as a standalone question and when embedded. We think that the difference may be explained pragmatically. A disjunction of two questions expresses an issue that may be resolved equally well by providing information resolving the first disjunct, or by providing information resolving the second disjunct. Now, it is difficult to see what kind of motivation (or what kind of decision problem, to follow van Rooij 2003) a speaker could have that would lead her to raise or even consider the issue expressed by (13). This is very different in the case of (14): in this case, it is immediate to reconstruct the sort of motivation that may lead a speaker to consider the relevant issue. We suggest that the different cognitive plausibility of the two issues at stake underlies the difference in the perceived felicity of the associated questions.

9.2.3 Dynamic partition semantics

While Hamblin (1973), Karttunen (1977), and Groenendijk and Stokhof (1984) all operate under a static view on meaning, there are also a number of proposals that aim to capture the meaning of questions in a dynamic framework. The first such proposals, developed by Jäger (1996), Hulstijn (1997), and Groenendijk (1999), essentially reformulate the partition theory of questions in the format of an update semantics (Veltman, 1996). This means that they construe the meaning of a sentence as its context change potential, i.e., a function that maps a context to a new context. Just like we do here, these theories do not model a context simply as a set of worlds—embodifying the information established in the conversation so far—but provide a more refined notion of context, one that also embodies the issues that have been raised so far. More specifically, a context C is modeled as an equivalence relation over a set of worlds s ⊆ W. Such an equivalence relation, which induces a partition on s, can be taken to encode both information and issues. On the one hand, the information established in C is encoded by the set of all worlds that are in the domain of C, i.e., all worlds in s. On the other hand, the issues present in C are encoded by the partition that C induces: two worlds w and v are connected by C and therefore included in the same partition cell just in case the distinction between w and v is not (yet) at stake in the conversation. In other words, C is conceived of as a relation encoding indifference (Hulstijn, 1997):

\[ \textit{indifference} \] (Hulstijn, 1997)
$w$ and $v$ are connected by $C$, the discourse participants have not yet expressed an interest in information that would distinguish between $w$ and $v$.

Both statements and questions can then be taken to have the potential to change the context in which they are uttered. A statement restricts the domain $s$ to those worlds in which the sentence is true (strictly speaking, it removes all pairs of worlds $\langle w, v \rangle$ from $C$ such that the sentence is false in at least one of the two worlds). Questions disconnect worlds, i.e., they remove a pair $\langle w, v \rangle$ from $C$ just in case the true exhaustive answer to the question in $w$ differs from the true exhaustive answer to the question in $v$.

Thus, the dynamic systems of Jäger (1996), Hulstijn (1997), and Groenendijk (1999) provide a notion of context and meaning that embodies both information and issues in an integrated way, in terms of an equivalence relation encoding indifference. However, just as classical partition semantics, these dynamic systems are too restrictive to allow for a satisfactory treatment of conditional questions, disjunctive questions, and mention-some $wh$-questions.\(^\text{17}\)

### 9.3 Inquisitive indifference semantics

A core assumption of dynamic partition semantics is that indifference should be encoded by means of an equivalence relation between possible worlds. Mascarenhas (2009) and Groenendijk (2009) suggest that the limitations of the framework may be overcome by re-examining this assumption. Of course, whether indifference should be encoded by means of an equivalence relation between possible worlds depends on how exactly one conceives of the notion of indifference. One natural perspective is that an agent is indifferent between two worlds $w$ and $v$ just in case $w$ and $v$ agree on the truth value of all propositions that the agent deems relevant. Under this perspective, a relation encoding the agent's indifference should indeed be an equivalence relation—it should be reflexive because every world will clearly agree with itself on the truth value of all relevant propositions; it should be symmetric because 

\(^{17}\) At least not without further amendments. Isaacs and Rawlins (2008) develop a dynamic partition semantics that allows for hypothetical updates of the context of evaluation. This framework allows for a natural analysis of conditional questions. However, open disjunctive questions and mention-some $wh$-questions remain beyond its reach.
agrees with \( v \) then \( v \) must also agree with \( w \); and it should be transitive because if \( w \) agrees with \( v \) and \( v \) with \( u \), then \( w \) must agree with \( u \) as well.

However, this is not the only sensible way to conceive of the notion of indifference. Another natural perspective is that an agent is indifferent between two worlds \( w \) and \( v \) just in case, if she were to be given the information that the actual world is either \( w \) or \( v \), the issues that she entertains would be resolved and she would not require further information determining precisely which of \( w \) and \( v \) is the actual world. Under this perspective, indifference relations should still be reflexive and symmetric, but they do not necessarily have to be transitive. To see this, suppose that our agent entertains just one issue, namely the one expressed by the mention-some question in (19):

\[
(19) \quad \text{Where can one buy an Italian newspaper?}
\]

Now consider the following three possible worlds:

- \( w_1 \): Italian newspapers are only sold at Central Station,
- \( w_2 \): Italian newspapers are only sold at the airport,
- \( w_3 \): Italian newspapers are sold in both places.

If the agent were to be told that the actual world is either \( w_1 \) or \( w_3 \), her issue would be resolved: she could go get her newspaper at Central Station and would not require further information determining which of \( w_1 \) and \( w_3 \) is the actual world. The same holds if the agent were to be told that the actual world is either \( w_2 \) or \( w_3 \); in this case she could get her newspaper at the airport. However, if she were told that the actual world is either \( w_1 \) or \( w_2 \), then she would need further information in order to decide where to go: if the actual world is \( w_1 \) she has to go to Central Station, but if it is \( w_2 \) she has to go to the airport. Thus, the agent is indifferent between \( w_1 \) and \( w_3 \) as well as between \( w_3 \) and \( w_2 \), but not between \( w_1 \) and \( w_3 \). This means that her indifference relation is not transitive.

In view of such considerations, Groenendijk (2009) and Mascarenhas (2009) dropped the transitivity constraint on indifference relations. In the resulting framework, which they referred to as inquisitive semantics, the alternatives associated with a question are maximal sets of worlds such that each pair in the set stands in the indifference relation induced by the question. Since indifference relations are no longer required to be transitive, the alternatives associated with a question may overlap. For instance, in the above example, if we take our logical space to consist
of $w_1$, $w_2$ and $w_3$, the alternatives associated with the question in (19) are \{w_1, w_3\} and \{w_2, w_3\}. These alternatives overlap, since they both contain $w_3$. Note that \{w_1, w_2, w_3\} is not an alternative associated with (19), because $w_1$ and $w_2$ do not stand in the indifference relation that (19) induces.

Allowing for issues with overlapping alternatives by dropping the transitivity requirement on indifference relations makes it possible to extend the scope of partition semantics. However, Ciardelli (2008) observed that the gain in generality resulting from this move is not yet sufficient. While conditional questions like (12a) and open disjunctive questions with two disjuncts like (12c) can be dealt with satisfactorily, disjunctive questions with three or more disjuncts remain problematic, and the same goes for mention-some wh-questions like (19).

To briefly illustrate what the problem is (see also Exercise 9.5), consider a context $c$ consisting of three possible worlds, as above, but now let the availability of Italian newspapers in these worlds be as follows:

- $w_1$: Italian newspapers are only sold at the Central Station and the zoo.
- $w_2$: Italian newspapers are only sold at the airport and the zoo.
- $w_3$: Italian newspapers are only sold at the Central Station and the airport.

The question in (19) is not resolved in this context, since $c$ does not imply of any place that it sells Italian newspapers. However, for any pair of worlds $v, v'$ from the context $c$, if we are given the information that the actual world is either $v$ or $v'$, then more precise information distinguishing between the two worlds is not needed in order to resolve (19). Thus, the indifference relation expressed by (19) in the given context is the total relation—which amounts to a trivial issue. This shows that the approach of Groenendijk (2009) and Mascarenhas (2009) cannot detect the fact that (19) expresses a non-trivial issue in the given context.

This example shows that the resolution conditions of a question cannot in general be reconstructed from the indifference relation that it induces. Moreover, Ciardelli (2008) argued that it is impossible to overcome this problem without letting go of the most fundamental notion in the framework of Groenendijk and Mascarenhas, inherited from dynamic partition semantics, namely that of issues encoded by means
of indifference relations. This insight led to the inquisitive semantics framework presented in this book.\(^{18}\)

To distinguish the two stages in the development of inquisitive semantics, we refer to the framework proposed by Groenendijk (2009) and Mascarenhas (2009) as inquisitive indifference semantics.\(^{19}\) In terms of restrictiveness, inquisitive indifference semantics is situated in between partition semantics and the current inquisitive semantics framework, as indicated in Figure 9.1. This is a direct consequence of the main commonalities and differences between these frameworks. On the one hand, inquisitive indifference semantics is similar to dynamic partition semantics and differs from the present inquisitive semantics framework in that it encodes issues by means of indifference relations. On the other hand, it is similar to the present inquisitive semantics framework and different from dynamic partition semantics in that it ultimately characterizes issues in terms of what is needed to resolve them. Only, unlike in the present framework, this is not done directly, but via indifference relations. That is, an issue is encoded as an indifference relation, and whether this relation holds between two worlds \(w\) and \(v\) depends on whether the issue is resolved by the information that the actual world is either \(w\) or \(v\). In the present framework, the connection between issues and resolution conditions is more direct, since indifference relations no longer play a role.

Summing up, while some important aspects of inquisitive indifference semantics persist in the present framework, its core notion—issues encoded by means of indifference relations—has been replaced by a more general one, and this generality is needed to suitably capture the full range of question types in natural languages. Thus, our framework naturally fits within the existing tradition of semantic theories of informative and inquisitive discourse, but it is more general and able to cover more empirical ground than its predecessors.

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18 A bit more historical detail: Groenendijk’s 2009 paper was written and started circulating in 2007, but appeared in print only in 2009. Mascarenhas’s 2009 master thesis was also largely written in 2007, but only presented in its final form in 2009. Ciardelli’s 2008 term paper was written in the fall of 2008, for a course taught by Groenendijk. The arguments presented in the term paper were further elaborated in Ciardelli (2009) and Ciardelli and Roelofsen (2011).

19 In previous work (e.g., Ciardelli, 2009; Ciardelli and Roelofsen, 2011), inquisitive indifference semantics has also been referred to as ‘inquisitive pair semantics’, since it can be characterized in terms of a notion of support with regard to pairs of worlds (see Groenendijk, 2009). Here, we instead use the term ‘inquisitive indifference semantics’ because it refers more transparently to the framework’s central concept.
9.4 Division of labor

So far, we have compared a number of approaches based on how they model question meanings—i.e., in terms of possible answers, true exhaustive answers, indifference relations, or resolution conditions. In this final section, we shift our attention to a different issue: What should the role of a compositional semantics be within a larger theory of question interpretation? That is, how should the labor be divided between the compositional semantics of questions and other components of the theory? We will first consider the received view on this issue, and then compare it to the one taken in inquisitive semantics.20

9.4.1 The received view

The received view is one which, in fact, assigns a very minimal role to compositional semantics. In order to say more specifically what this role is, we have to look separately at matrix questions and embedded questions. In the case of matrix questions, it is assumed under this view that the issue raised in asking the question is not only determined by the compositionally derived semantic value of the question, but also in part by a specific update rule associated with question speech acts.

Similarly, in the case of embedded questions, it is assumed that the semantic contribution of the embedded clause is not only determined by the compositionally derived semantic value of the question, but also by an answer operator which is assumed to always accompany embedded questions.

Let us make this more concrete by briefly outlining a particular theory instantiating this view. We opt here for the theory presented in Heim (2016), which is very explicit about the interaction between the semantics of questions, update rules, and answer operators. But many other contemporary theories of questions assume a similar division of labor and would in principle serve our present purposes equally well.

First, let us exemplify the semantic values that Heim (2016) compositionally assigns to simple wh-questions and polar questions. The wh-question in (20a) receives the semantic value in (20b), a set containing the classical proposition \(|Pd|\) for every individual \(d\).

(20)  a. Who is going to the party?
   b. \{ |Pd| : d \in D\}

20 The general argument made in this subsection is drawn from Farkas and Roelofsen (2017).
On the other hand, the polar question in (21a) receives the semantic value in (21b), a singleton set containing just $|P_j|$.

\begin{align*}
\text{a. Is John going to the party?} \\
\text{b. } \{ |P_j| \}
\end{align*}

Note that Heim operates in the alternative semantics framework of Hamblin (1973) and Karttunen (1977), where question meanings are arbitrary sets of classical propositions, i.e., they need not be downward closed, and they need not form partitions either. However, the semantic values that Heim derives, in particular for polar questions, do not coincide with those assumed by either Hamblin or Karttunen. That is, while Hamblin and Karttunen take a polar question like (21a) to have two ‘possible answers,’ $|P_j|$ and $|\neg P_j|$, the semantic value of (21a) in Heim’s system only contains the first of these two.

This should not be taken to reflect a disagreement as to what the ‘possible answers’ to polar questions like (21a) are. Rather, it reflects a different take on what the semantic value of a question is intended to represent. For Heim, the compositionally derived semantic value of a question is not intended to directly embody the set of ‘possible answers’ to that question, or any other type of answers for that matter. The semantic value of a question is just an abstract object that serves as input for the update rule associated with questions (in the case of matrix questions) or a suitable answer operator (in the case of embedded questions). We now turn to these ingredients of the theory.

In line with much earlier work, Heim assumes that in making a statement or asking a question, a speaker proposes to update the conversational context in a particular way. What the proposed update is, is determined by the update rules associated with statements and questions, respectively. To spell out what Heim takes these rules to be, we first need to briefly review how she models conversational contexts. In this, Heim essentially follows the work on dynamic partition semantics discussed above (Jäger, 1996; Hulstijn, 1997; Groenendijk, 1999). That is, she models a conversational context $C$ as a set of mutually disjoint classical propositions. The union of these classical propositions, $\bigcup C$, is a set of worlds, embodying the information that has been commonly established among the conversational participants in $C$. On the other hand, the elements of $C$ together form a partition over $\bigcup C$, embodying the exhaustive answers to the current question under discussion.
With this notion of contexts in place, Heim defines the following update rules for statements and questions, respectively.

**Definition 9.1 (Heim’s update rule for statements)**
In making a statement, i.e., in uttering a declarative sentence $\varphi$, a speaker proposes to replace the current context $C$ by a new context which is constructed by collecting all non-empty intersections of elements of $C$ with $|\varphi|$, the classical proposition expressed by $\varphi$. That is, $C$ is to be replaced by:

- $\{p \cap |\varphi| \mid p \in C \text{ and } p \cap |\varphi| \neq \emptyset\}$

**Definition 9.2 (Heim’s update rule for questions)**
In asking a question, i.e., in uttering an interrogative sentence $\varphi$, a speaker proposes to replace the current context $C$ by a new context which is constructed by re-partitioning $C$ into cells consisting of worlds that agree on the truth of every element of $[\varphi]$. That is, $C$ is to be replaced by:

- $\{\{v \mid \forall p \in [\varphi]: w \in p \iff v \in p\} \mid w \in \bigcup C\}$

To illustrate these update rules, suppose that our initial context is one in which no information has been established and no questions have been asked yet, i.e., $C = \{W\}$. Now suppose a speaker utters the declarative sentence *Mary is going to the party*, translated as $Pm$. Then, according to the update rule for statements in Definition 9.1, the speaker proposes to replace $C$ with the following context:

$$C' = \{p \cap |Pm| \mid p \in \{W\} \text{ and } p \cap |Pm| \neq \emptyset\}$$
$$= \{|Pm|\}$$

If the proposal is accepted by the other conversational participants, $C'$ becomes the new context, which means that the information that Mary is going to the party becomes common ground, and no further information is requested.

Now let us return to our initial context $C$ and suppose that the speaker instead utters the polar interrogative in (21a), *Is John going to the party?*. This time, according to the update rule for questions in Definition 9.2, the speaker proposes to replace $C$ with the following context:

$$C' = \{v \mid w \in |Pj| \iff v \in |Pj|\} | w \in W\}$$
$$= \{|Pj|, |\neg Pj|\}$$
Thus, no new information becomes common ground, since $\bigcup C'$ still covers the set of all possible worlds $W$. However, the context is now partitioned into two cells, $|Pj|$ and $|\neg Pj|$, which means that the participants become publicly committed to resolving the question whether John is going to the party or not.\(^{21}\)

Now let us move from matrix questions to embedded ones. Heim mainly focuses on questions embedded under *know*. For sentences in which *know* takes a declarative complement, she assumes the standard analysis. For instance, (24a) is assigned the truth-conditions in (24b):\(^{22}\)

(24) a. Ann knows that John is going to the party.
    b. $w \models K_a(Pj) \iff \sigma_a(w) \subseteq |Pj|

Here, $\sigma_a(w)$ is Ann’s information state in $w$, modeled as a set of possible worlds, and $|Pj|$ is the classical proposition expressed by $Pj$, also a set of possible worlds.

What if *know* takes a question as its complement, whose semantic value is not a set of possible worlds but rather a set of classical propositions? Clearly, the standard analysis of *know*, exemplified in (4), cannot immediately be applied in this case. To overcome this obstacle, Heim assumes, as many other authors have done as well, that the semantic value of an embedded question, before it combines with that of the verb, is first transformed by a so-called answer operator, $\text{ANS}$. This answer operator takes as its input a possible world $w$ and a question meaning $Q$, i.e., a set of classical propositions, and delivers as its output a single classical proposition $\text{ANS}(w,Q)$. More specifically, Heim defines the answer operator in such a way that $\text{ANS}(w,Q)$ is always the true exhaustive answer to $Q$ in $w$.

**Definition 9.3** (Heim’s answer operator)
For any possible world $w$ and any question meaning $Q$:

- $\text{ANS}(w,Q) := \{v | \forall p \in Q : v \in p \iff w \in p\}$

Thus, a sentence like (25a) receives the truth-conditions in (25b), which is a satisfactory result.

\(^{21}\) The reader is invited to verify that the partition induced by a wh-question like (20a) according to Heim’s update rule for questions is precisely the one that is associated with it in partition semantics.

\(^{22}\) We simplify here somewhat, leaving factivity presuppositions out of consideration. These are orthogonal to the main issues that we will be concerned with in this section.
(25) a. Ann knows whether John is going to the party.
   b. $w \models K_a?Pj \iff \sigma_a(w) \subseteq \text{ANS}(w, \{|Pj|, |\neg Pj|\})$

This completes our illustration of the received view on the division of labor between compositional semantics and other components of an overall theory of question interpretation. Note that, as indicated at the outset, the role assigned to compositional semantics on this view is very minimal. In the case of matrix questions a decisive role is played by the update rule, and in the case of embedded questions such a role is fulfilled by the answer operator. In both cases, the semantic value that is produced by the compositional semantics only serves as ‘raw material’ for these operators.

This, in our view, is a substantial weakness of the approach: it leaves the compositional semantics of questions highly unconstrained. That is, which values are produced by the compositional semantics does not matter all that much, as long as one can formulate an update rule and an answer operator which, when given these values as input, yield the desired output.

It is important to note in this regard that both the update rule and the answer operator are assumed to be specific to questions. Thus, they can be tailor-made for the purpose of transforming the compositionally derived semantic values to yield the desired output. They do not serve a broader purpose in the overall theory of interpretation, and are therefore not independently constrained. This diminishes the explanatory value of the approach.

It would be preferable to have a theory that does without any question-specific update rule or answer operator, i.e., one in which there is just one simple update rule that applies uniformly to statements and questions, and one in which the embedding operator—if at all present—is not specific to questions, but applies uniformly to declarative and interrogative embedded clauses. This is precisely the kind of approach that comes naturally with inquisitive semantics.

9.4.2 The inquisitive perspective

Let us first consider matrix questions and then embedded ones. We have already seen in Section 2.5.3 that inquisitive semantics comes with a natural notion of context update, which is simply defined in terms of
set intersection and applies uniformly to statements and questions. This allows for a unification and simplification of Heim’s update rules for statements and questions.

**Definition 9.4** (Inquisitive update rule for statements and questions)
In uttering a sentence \( \varphi \), be it a declarative or an interrogative, a speaker proposes to replace the context \( C \) by a new context which is constructed by intersecting \( C \) with \( [\varphi] \). That is, \( C \) is to be replaced by \( C \cap [\varphi] \).

For comparison, let us apply this rule to the examples considered above. As before, let the initial context \( C \) be one in which no information is available and no issues have been raised yet. In inquisitive semantics, this context is represented as \( \{W\} \). Now suppose a speaker utters the declarative sentence *Mary is going to the party*, translated as \( Pm \). Then, according to our general update rule, the speaker proposes to replace \( C \) with the following context:

\[
C' = C \cap [Pm] \\
= \{W\} \cap \{|Pm|\} \\
= \{|Pm|\}
\]

If the proposal is accepted by the other conversational participants, \( C' \) becomes the new context, which means that the information that *Mary is going to the party* becomes common ground, and no further information is requested.

Now let us again return to our initial context \( C \) and suppose that the speaker instead utters the polar interrogative in (21a), *Is John going to the party?* According to our general update rule, the speaker proposes to replace \( C \) with the following context:

\[
C' = C \cap [?Pj] \\
= \{W\} \cap \{|Pj|, \neg Pj\} \\
= \{|Pj|, \neg Pj\}
\]

In this case, the speaker does not provide any information herself, but she does raise an issue, requesting information from other participants in order to establish a common ground that is contained either in \( |Pj| \) or in \( \neg Pj \).

Thus, the results are essentially the same as in Heim’s system, but they are obtained by using a uniform update rule (indeed, the standard intersective update rule) rather than two separate update rules for statements and questions.
It should be emphasized that what we have gained is not just simplicity. More importantly, the fact that the update rule determining the contextual effect of questions is not question-specific but rather plays a more general role in the overall theory means that it is constrained by considerations that are independent of questions altogether. This means that the approach leaves less room for ad-hoc stipulations, and is therefore more explanatory.

Now let us turn to embedded questions. We have seen in Section 8.2.2 that in inquisitive epistemic logic the standard analysis of know is generalized in such a way that it can deal uniformly with both declarative and interrogative complements. The support conditions for $K_a \varphi$ are repeated in (28) and the truth-conditions which can be derived from this in (29).

\[(28) \quad s \models K_a \varphi \iff \forall w \in s : \sigma_a(w) \models \varphi\]
\[(29) \quad w \models K_a \varphi \iff \sigma_a(w) \models \varphi\]

Applying this analysis to the examples considered above yields the following results:\(^{23}\)

\[(30) \quad \text{a. } Ann \text{ knows that John is going to the party. }\]
\[\quad \text{b. } w \models K_a(Pj) \iff \sigma_a(w) \models Pj \iff \sigma_a(w) \subseteq |Pj|\]

\[(31) \quad \text{a. } Ann \text{ knows whether John is going to the party. }\]
\[\quad \text{b. } w \models K_a ?Pj \iff \sigma_a(w) \models ?Pj \iff \begin{cases} \sigma_a(w) \subseteq |Pj| & \text{if } w \in |Pj| \\ \sigma_a(w) \subseteq |\neg Pj| & \text{if } w \in |\neg Pj| \end{cases}\]

Thus, again, the same results are obtained as in Heim’s account, but this time without an answer operator. Just as in the case of matrix questions, this is not just a gain in simplicity, but also in explanatory force—assuming an answer operator that is specific to embedded questions and does not serve a more general purpose in the overall theory makes room for ad-hoc customization and leaves the compositional semantics of questions highly unconstrained. On the other hand, doing without such an answer operator leads to a theory in which the compositional semantics of questions has to deliver semantic values which can be fed immediately to the embedding verbs, without any transformation. This, together with the fact that the same semantic values should also serve

\(^{23}\) The derivation in (31b) assumes that $\sigma_a$ is factive, i.e., that for any $w$, $w \in \sigma_a(w)$. For discussion of this constraint, see page 150.
as input to the general intersective update rule discussed above in case the question occurs in matrix form, results in a much more constrained theory, and thus one with a greater explanatory value.  

9.5 Exercises

**Exercise 9.1 Inquisitive semantics and alternative semantics**

1. What is the technical difference between question meanings in alternative semantics and in inquisitive semantics?

2. Conceptually, what is the reason for this difference?

3. What are the repercussions of this difference for the analysis of questions in natural languages?

**Exercise 9.2 Conjunction in alternative semantics**

Assume that in alternative semantics a polar question $?p$ is associated with the set of possible answers $[?p] = \{[p], \overline{[p]}\}$.

1. Suppose conjunction is analysed in terms of intersection:

$$[Q \land Q'] = [Q] \cap [Q']$$

What set of possible answers does this analysis yield for (32)?

(32) Is the concert today, and will you attend it?

2. Now suppose conjunction is analysed in terms of point-wise intersection:

$$[Q \land Q'] = \{a \cap a' \mid a \in Q \text{ and } a' \in Q'\}$$

What set of possible answers does this analysis deliver for (32)?

3. Can this point-wise conjunction operation be characterized as a meet operator with respect to some relation of entailment?

*Hint:* a meet operation must validate certain principles: in particular, it must be commutative ($a \land b = b \land a$), associative ($a \land (b \land c) = (a \land b) \land c$), and idempotent ($a \land a = a$). Does the above definition of conjunction validate these principles?

---

24 A detailed inquisitive account of declarative and interrogative embedded clauses and the verbs that take such clauses as their complement is given in Theiler et al. (2016b). This account, unlike the one presented here, assumes that embedded clauses always involve a so-called *embedding operator*. This operator, however, is not specific to embedded questions. It applies uniformly to both declarative and interrogative embedded clauses. Thus, it does not make the overall theory less constrained.
Exercise 9.3  Inquisitive semantics and partition semantics

Let $I$ be an issue, $w$ a world, and $a$ a classical proposition. We say that $a$ is a true complete answer to $I$ at $w$ if (i) $w \in a$ and (ii) for every information state $s$ with $w \in s: s \subseteq l \iff s \subseteq a$.

1. Show that if a true complete answer to $I$ at $w$ exists, then it is unique.

2. Let us say that an issue $I$ is a partition issue if it is induced by a partition, i.e., if there is a partition $\rho$ such that $I = I_\rho$, where $I_\rho := \{s \subseteq t | t \in \rho\}$. Show that $I$ is a partition issue iff a true complete answer to $I$ exists at each possible world.

3. Show that if an issue $I$ has two overlapping alternatives, or its alternatives do not cover the whole logical space, then $I$ is not a partition issue.

Exercise 9.4  Approximate value questions

Consider the following question:

(33) How many stars are there in the Milky Way, give or take ten?

1. Describe the issue expressed by this question.

2. Using the characterization of partition issues given in the previous exercise, show that this issue is not induced by a partition of the logical space.

Exercise 9.5  Inquisitive semantics versus indifference semantics

Determine the interpretation of a disjunction with three disjuncts, $p \lor q \lor r$, in the semantics of Groenendijk (2009) and Mascarenhas (2009). How does this differ from the proposition assigned to $p \lor q \lor r$ in InqB? How does this difference arise?
Conclusion

We will end with an overview of the main concepts that play a role in the framework we presented, emphasizing its modular architecture. After that, we will return to the high-level desiderata discussed in Chapter 4 and consider to what extent they have been met.

10.1 Overview of main concepts

Figure 10.1 provides an overview of the main concepts that play a role in InqB, the basic inquisitive semantics system presented in Chapter 4, and the dependencies between them. InqB assumes a particular logical language $L$, namely the language of standard first-order logic (the upper leftmost item in the diagram). Given this language, we defined the models relative to which the sentences in our language would be interpreted. A model was construed as a set of possible worlds $W$, associated with a domain of discourse and an interpretation function determining the denotation of the basic elements of our language (function symbols and relation symbols) in each possible world. Thus, a model determines a certain logical space, the set of worlds $W$, as well as a particular connection between the worlds in this space and the basic elements of the language under consideration.

We adopted the standard notion of information states as sets of possible worlds, i.e., subsets of $W$. In terms of information states, we defined a new notion of issues, and based on this notion of issues we introduced a notion of propositions encompassing both informative and inquisitive content. We defined a notion of entailment between propositions, and characterized two kinds of semantic operators on propositions: (i) algebraic operators, which for instance yield the meet or the join of two propositions with regard to entailment, and (ii) projection operators, which trivialize either the informative or the inquisitive content of a given proposition. Finally, based on these semantic operators,
we defined a semantics for the language $\mathcal{L}$ that we started out with, coming full circle.

Having laid out this schematic overview of InqB, we would like to emphasize that all the notions which play a crucial role in this system, except for the logical language and the models with respect to which the sentences in the language are interpreted, were already characterized in Chapters 2–3, without reference to any particular logical or natural language. This makes these notions highly general and widely applicable.

As we saw in Chapter 4, what becomes necessary when turning to a particular language is a more specific characterization of the assumed logical space. In Chapters 2–3, we just assumed a generic set of possible worlds $W$ as our logical space, without any further specification. The moment we fix a particular logical language, we have to establish a connection between the worlds in our logical space and the basic elements of our language. Thus, in Chapter 4, we supplemented the set of possible worlds $W$ with a domain of individuals $D$ and a function $I$ determining the denotation of the basic elements of our language (in this case, function symbols and relation symbols) with regard to each world $w \in W$. Having fixed this connection between ‘worlds and words’, all the general notions introduced in Chapters 2–3 could be imported straightforwardly.

In Chapter 8 we considered an extension of InqB with modal operators to describe the knowledge and issues of a given set of agents. Accordingly, we equipped our logical space with a set of state maps $\Sigma_A$, determining the information states and inquisitive states of all the agents at every world. Having thus equipped the possible worlds in our logical
space with the structure needed to interpret our extended language, all the general notions laid out in Chapters 2–3 could once again be imported straightforwardly.

The fact that the framework is built up in this modular way makes it very flexible. There are many ways in which the basic notions introduced here may be further refined, extended, and applied (see Further Reading for some references).

10.2 Mission accomplished?

Let us now return to the high-level desiderata discussed in Section 1.2, and assess to what extent the framework we presented addresses these desiderata.

The first high-level desideratum was a formal notion of issues that allows for a suitable representation of semantic content, conversational contexts, and propositional attitudes. In Chapter 2 we introduced such a notion of issues, and in terms of it we defined new notions of semantic content (propositions), conversational contexts, and context update. In Chapters 5, 6, and 9 we argued that the new notion of semantic content is particularly suitable for the analysis of questions, overcoming the main shortcomings of previous frameworks for question semantics (alternative semantics, partition semantics, and indifference semantics). In Chapter 8 we showed that the new notion of issues facilitates a richer view on propositional attitudes as well, encompassing both information-directed attitudes like know and issue-directed attitudes like wonder.

The second high-level desideratum was a framework that allows for an integrated treatment of declarative and interrogative sentences, with a single notion of semantic content which is general enough to deal with both sentence types at once, rather than a separate notion of content for each sentence type. One argument we made to justify this desideratum was that declarative and interrogative sentences are to a large extent built up from the same lexical, morphological, and intonational elements. A general characterization of the semantic contribution of each of these elements should capture both their contribution to the informative content and to the inquisitive content of the sentence that they are part of. This requires a framework in which the semantic content of a sentence—the proposition it expresses—encompasses both informative and inquisitive content.
The notion of propositions introduced in Chapter 2 satisfies this requirement, and the merits of this feature of the framework were illustrated most explicitly in Chapter 6 with an analysis of declarative and interrogative sentences involving disjunction and various intonation patterns. Both disjunction and the relevant intonational elements were given a uniform treatment across various sentence types. Another important result of the approach was discussed in Chapter 7: while originally intended to broaden the domain of logical semantics from declaratives to interrogatives, we have seen that it also leads to an improved analysis of declaratives as such. We illustrated this point in the domain of conditionals, whose truth-conditions are sensitive to the inquisitive content of their antecedent.

Thus, both desiderata have been met and the ensuing benefits have been concretely substantiated. From a narrow perspective, then, our goals have been achieved. From a broader perspective, however, these results just indicate that our general mission is worthwhile pursuing. We do not see the basic framework presented here as a final product but much rather as a point of departure.
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Further reading

Despite its relatively recent inception, there has already been a lot of work on inquisitive semantics, much more than we have been able to cover in this book. The basic framework presented here has been further extended, refined, and applied in several ways, the logical properties of the framework have been investigated, and some interesting connections with other logical frameworks have emerged, though in all these areas there are still many open issues to be addressed. Below we provide some pointers for further reading.

Extensions of InqB and IEL

- A type-theoretic extension of InqB, for full compositionality: Ciardelli, Roelofsen, and Theiler (2017a)
- An extension of InqB with presuppositions: Ciardelli, Groenendijk, and Roelofsen (2012, 2015); Roelofsen (2015a)
- An extension of InqB with propositional discourse referents: Roelofsen and Farkas (2015)
- Integration of InqB with a commitment-based discourse model: Farkas and Roelofsen (2017)
- An extension of IEL with dynamic operators that model the effects of statements and questions that are publicly observable by all conversational participants: Ciardelli and Roelofsen (2015)
- An extension of IEL with dynamic operators that model the effects of statements and questions that may only be partially observable by some conversational participants: van Gessel (2016)
- An extension of IEL with graded beliefs next to hard knowledge: Ciardelli and Roelofsen (2014); Sparkes (2016)
- An extension of InqB with a weak negation operator, whose treatment requires the existence of propositions that are not downward closed: Punčochář (2015)

Generalizations and refinements of InqB

- Generalizations of InqB based on non-classical logics of statements: Punčochář (2016b,a, 2017); Ciardelli et al. (2017b)
- A refinement of InqB that is not only concerned with informative and inquisitive content, but also 'attentive content', whose treatment again
requires propositions that are not downward closed:¹
Ciardelli, Groenendijk, and Roelofsen (2014)
• A refinement of InqB that is not only concerned with informative and inquisitive content, but also with ‘live possibilities’, those possibilities that are to be taken seriously in inquiry:
Roelofsen (2016) (see also Willer, 2013, 2017, for closely related work)
• A refinement of InqB that does not characterize a proposition just in terms of the states that support it, but also in terms of the states that reject it or ‘dismiss a supposition’ of it, referred to as InqS:
Groenendijk and Roelofsen (2015)
• An extension of InqS with operators corresponding to epistemic and deontic modal auxiliaries (*might, may, must*):
Aher and Groenendijk (2015)

Logical investigations
• Logical investigation of InqB:
Ciardelli (2009); Ciardelli and Roelofsen (2011); Ciardelli (2016d); Grilletti (2017); Grilletti and Ciardelli (2017)
• Logical investigation of IEL:
Ciardelli (2014, 2016d); Ciardelli and Otto (2017)
• Logical investigation of extensions and generalizations of InqB:
Punčochář (2015, 2016b,a, 2017); Ciardelli et al. (2017b)
• Logical investigation of inquisitive indifference semantics:
Mascarenhas (2009); Sano (2009, 2011)

Applications in linguistics
• Root questions:
AnderBois (2011, 2012); Champollion et al. (2015); Roelofsen and Farkas (2015); Roelofsen (2015a); Farkas and Roelofsen (2017)
• Embedded questions and question-embedding verbs:
Theiler (2014); Theiler et al. (2017); Herbstritt (2014); Roelofsen et al. (2016); Roelofsen (2017); Roelofsen and Uegaki (2016); Cremers et al. (2017a); Ciardelli and Roelofsen (2018)
• Disjunction:
AnderBois (2011, 2012); Winans (2012); Roelofsen (2015a,b); Ciardelli and Roelofsen (2017a)

¹ There is also work which argues that it is in fact impossible to capture all three types of content—informative, inquisitive, and attentive—at once using a single semantic object, and pursues a two-dimensional approach instead (Roelofsen, 2013b).
• Conditionals:
  Onea and Steinbach (2012); Starr (2014); Groenendijk and Roelofsen (2015); Champollion et al. (2016); Ciardelli (2016b); Ciardelli et al. (2017c); Willer (2017)
• Modal auxiliaries:
  Aher (2013); Ciardelli et al. (2014); Aher and Groenendijk (2015); Willer (2015, 2017); Roelofsen (2013b, 2016)
• Scalar modifiers:
  Coppock and Brochhagen (2013a,b); Blok (2015); Ciardelli et al. (2016); Cremers et al. (2017b)
• Implicit questions in discourse: Onea (2013)
• Answer particles (yes/no): Roelofsen and Farkas (2015)
• Quantifier particles: Szabolcsi (2015b)
• Ellipsis: AnderBois (2014, 2016a)
• Exhaustivity implicatures: Westera (2012, 2013a,b, 2017)
• Imperatives: Aloni and Ciardelli (2013); Ciardelli and Aloni (2016)

Applications in cognitive science
• Reasoning fallacies: Koralus and Mascarenhas (2014); Mascarenhas (2014)
• Implicit causality: Spenader (2015)

Applications in philosophical logic and epistemology
• General perspective on the role of questions in logic: Ciardelli (2018, 2016d)
• Fatalism: Bledin (2017)
• The Gettier puzzle: Uegaki (2012)
• Conversational inquiry: Hamami (2014)
• Belief revision: Ciardelli and Roelofsen (2014); Sparkes (2016)
• Contrastive knowledge: Cohen (2017)

Related frameworks
• Dependence logic: Väänänen (2007)
  Discussion of connections with inquisitive semantics:
  Yang (2014); Yang and Väänänen (2016); Ciardelli (2016a,d)
• Truth-maker semantics: Fine (2014); Yablo (2014)
  Discussion of connections with inquisitive semantics: Ciardelli (2013)
• Possibility semantics for modal logic: Holliday (2014, 2018)
  Discussion of connections with inquisitive semantics: Ciardelli (2016d)
• Dynamic epistemic logic with questions: 
  Minică (2011); van Benthem and Minică (2012) 
  Discussion of connections with inquisitive semantics: 
  Ciardelli and Roelofsen (2015); Ciardelli (2016d); van Gessel (2016) 
• Knowing value logic: Wang and Fan (2013, 2014); Fan et al. (2015) 
  Discussion of connections with inquisitive semantics: Ciardelli (2016d) 
• Mental models theory: Johnson-Laird (1983) 
  Implementation using ideas from inquisitive semantics: 
  Mascarenhas (2014); Koralus and Mascarenhas (2014) 
• Inferential erotetic logic: Wiśniewski (1995) 
  Discussion of connections with inquisitive semantics: 
  Wiśniewski and Leszczyńska-Jasion (2015)
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